Volatility-of-Volatility Risk and Asset Prices

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Abstract

While investors are averse to high market volatility, there is possibility that high market volatility could fluctuate even further, which could drive investors to dump risky assets with high required returns compensated for the volatility uncertainty. In this paper, we develop an equilibrium model, in which, in addition to market beta and variance risk, the flight risk associated with uncertainty about market volatility (i.e., variance of market variance) affects asset pricing. To test our model, we use the high-frequency S&P 500 index option data to estimate a time series of the variance of market variance. Consistent with the model, we find that high volatility-of-volatility stocks (i.e., returns co-move more negatively with the variance of market variance) have higher expected returns. A hedge portfolio long in high volatility-of-volatility stocks and short in low volatility-of-volatility stocks yields a significant 10.5 percent average annual return. Furthermore, the volatility-of-volatility risk largely subsumes the valuation effect of volatility risk documented in previous studies. In sum, our model and test results provide a unified framework to better understand the importance of volatility-of-volatility risk in asset pricing.

JEL classification: G12, G13, E44

1. Introduction

It is well established that volatility is time-varying and tends to be high during stock market decline. The role of uncertainty during the recent financial crisis is also noted in financial press. For example,

> CRISES feed uncertainty. And uncertainty affects behaviour, which feeds the crisis. …all the indicators of uncertainty are at or near all-time highs. What is at work is not only objective, but also subjective uncertainty (e.g. the unknown unknowns).
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> —Olivier Blanchard, *The Economist*, January 29, 2009.

The dual volatility concept has important implications for asset prices. Time-varying volatility-of-volatility affects portfolio decisions by inducing changes in investment opportunity set; it changes the expectation of future market returns and future market volatility. If volatility-of-volatility is a state variable, the Intertemporal CAPM (ICAPM; Merton, 1973) posits that volatility-of-volatility should be a priced factor in the cross-section of stocks. Intuitively, assets that co-vary positively with volatility-of-volatility are attractive to investors since these assets provide hedge for volatility risk during the market downturns. Moreover, it has been well established that market volatility is a priced factor (e.g. Coval and Shumway, 2001; Ang, Hodrick, Xing, and Zhang, 2006; and Adrian and Rosenberg, 2008) and therefore an increase in volatility-of-volatility induces volatility shock, leading to an increased required return and immediate stock price decline. Thus, investors require a return premium for a security that is suffer when the market volatility is high and when the whole market is uncertain about uncertainty.

This paper develops a market-based three-factor model that helps explain how asset prices are affected by volatility risk and volatility-of-volatility risk. The model provides a unified framework that can explain the empirical findings that aggregate volatility risk is priced in cross-sectional stock returns (e.g. Ang, Hodrick, Xing, and Zhang, 2006), that variance beta is priced in cross-sectional variance risk premiums (e.g. Carr and Wu, 2009), and that individual variance risk premiums can predict the cross-sectional stock returns (e.g. Bali and Hovakimian, 2009; Han and Zhou, 2011).

Our model begins with a macroeconomic model that incorporates the seminal long-run risks (LRR) model of Bansal and Yaron (2004) and the variance-of-variance model of Bollerslev, Tauchen, and Zhou (2009). We solve the macro-finance model explicitly and derive the equilibrium aggregate prices. Then, we use the properties of the aggregate asset prices to characterize the macroeconomic risks, transforming the underlying macro-based model to a market-based model. The market-based model developed in this paper has several advantages. First, financial data provide useful information because asset prices tell us how market participants value risks. Moreover, financial data convey information to public in a timely fashion. Hence, the empirical design of our model is compatible with a large literature of multi-factor model explaining cross-sectional monthly stock returns (see, for instance, Fama and French, 1993; Ang, Hodrick, Xing, and Zhang, 2006; Maio and Santa-Clara, 2012; among others).

In our model, the expected stock return of a security i is determined by three sources of risks. These risks are associated with: (i) the return sensitivity to market return, $\mathbb{C}\mathrm{ov}_t[r_{i,t+1}, r_{m,t+1}]$; (ii) the return sensitivity to market variance, $\mathbb{C}\mathrm{ov}_t[r_{i,t+1}, V_{m,t+1}]$, where $V_{m,t} = \mathbb{V}\mathrm{ar}_t[r_{m,t+1}]$; and, (iii) the return sensitivity to variance of market variance, $\mathbb{C}\mathrm{ov}_t[r_{i,t+1}, Q_{m,t+1}]$, where $Q_{m,t} = \mathbb{V}\mathrm{ar}_t[V_{m,t+1}]$. The first term measures the market risk of classical capital asset pricing model (CAPM; Sharpe, 1964; Lintner, 1965). The second term corresponds to the aggregate volatility risk of Ang, Hodrick, Xing, and Zhang (2006). The last term, which is the main focus of this paper, measures the aggregate variance of variance risk. Hereafter the paper, we refer to the variance of variance as the volatility-of-volatility.

The first goal of this paper is to investigate how market volatility-of-volatility risk affects cross-sectional stock returns. We test the predictions of the model using NYSE, AMEX, and NASDSQ listed stocks over the period 1996 to 2010. To implement our model, we develop a measure of market volatility-of-volatility using high frequency S&P 500 index option data.[1] We convert the tick-by-tick option data to equally spaced five-minute observations and then use the model-free methodology[2] to estimate the market variance implied by index option prices for each five-minute interval. Thus, for each day, we estimate the market volatility-of-volatility by calculating the realized bipower variance from a series of five-minute model-free implied market variance within the day. The bipower variation, introduced by Bardorff-Nielsen and Shephard (2004), delivers a consistent estimator solely for the continuous component of the volatility-of-volatility whereas the jump component is isolated.[3] In other words, our empirical results are robust to the potential jump risk embedded in volatility (see, for example, Pan, 2002; Eraker, 2008; Drechsler and Yaron 2011; among others).

Consistent with the model, by sorting stocks into quintile portfolios based on return sensitivities to market volatility-of-volatility, we find that stocks in the highest quintile have lower stock returns than stocks in the lowest quintile by 0.88 percent per

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[1] We use the volatility index, VIX index, from the Chicago Board of Options Exchange (CBOE) as the proxy for the aggregate volatility risk, which has been shown to be a significant priced factor in the cross-sectional stock returns (e.g. Ang, Hodrick, Xing, and Zhang, 2006).

[2] It has been shown that the expectation of market variance can be inferred in a 'model-free' fashion from a collection of option prices without the use of a specific pricing model (see, for example, Carr and Madan 1998; Britten-Jones and Neuberger 2000; Bakshi, Kapadia, and Madan, 2003; Jiang and Tian 2005). The option implied information is forward-looking and the estimate can be obtained using daily or intraday option data.

[3] Measures of realized jump based on the difference between realized variation and bipower variation have been proposed by Barndorff-Nielsen and Shephard (2004), Huang and Tauchen (2005), and Andersen, Bollerslev, and Diebold (2007).

month. Moreover, we also find evidence consistent with Ang, Hodrick, Xing, and Zhang (2006)'s findings that there is a significant difference of -0.87 percent per month between the stock returns with high volatility risk and the stocks with low volatility risk. Controlling for volatility risk, we still find that the market volatility-of-volatility carries a statistically significant return differentials of -0.97 percent per month. On the other hand, controlling for market volatility-of-volatility risk, we find the return difference between high volatility risk stocks and low volatility risk stocks is still large in magnitude, at -0.68 percent per month. Running the cross-sectional regressions, we find that market volatility-of-volatility carry a statistically significant negative price of risk and largely subsumes the valuation effect of volatility risk. Thus, our findings suggest that market volatility-of-volatility is indeed an independently priced risk factor in the cross-sectional stock returns.

To further explore the mechanism that volatility-of-volatility risk affects asset prices, we investigate whether the volatility-of-volatility risk contributes to the asymmetric correlations between returns and market volatility-of-volatility. We refer to the volatility-of-volatility feedback effect as the mechanism that if volatility-of-volatility is priced, an anticipated increase in volatility-of-volatility raises the required rate of return, implying an immediate stock price decline and higher future returns.[4] Consistent with the channel of volatility-of-volatility feedback effect, we find that stocks that co-move more negatively with market volatility-of-volatility have lower returns before the portfolio formation and earn higher post-formation returns than stocks that co-move more positively. More importantly, we find that the return differentials (e.g. the returns of negative exposure stocks minus the returns of positive exposure stocks) before the portfolio formation are negatively correlated with

[4] Our definition of volatility-of-volatility feedback effect follows the definition of volatility feedback effect in the literature (see, e.g. French, Schwert, and Stambaugh 1987; Campbell and Hentschel 1992; Bekaert and Wu 2000; Wu 2001; Bollerslev, Sizova, and Tauchen, 2012; among others).

market volatility-of-volatility measured at the portfolio formation date while the correlations between market volatility-of-volatility and the post-formation return differentials are positive. Hence, market volatility-of-volatility seems to be the state variable that drives the feedback effect, supporting the time-varying risk premium hypothesis.

The second goal of this paper is to investigate how volatility-of-volatility risk affects cross-sectional variance risk premiums. The variance risk premium is defined as the difference between risk-neutral variance and realized variance. Define $V_{i,t}$ as the conditional variance of stock i at time t, $V_{i,t} = \mathbb{V}\mathrm{ar}_t\left[r_{i,t+1}\right]$. In our model, the variance risk premium of stock i , $VRP_{i,t} \equiv \mathbb{E}_t^{\mathbb{Q}}\left[V_{i,t+1}\right] - \mathbb{E}_t\left[V_{i,t+1}\right]$, is determined by two sources of risks: (i) the variance sensitivity to market variance, $\mathbb{C}\mathrm{ov}_t\left[V_{i,t+1}, V_{m,t+1}\right]$; and, (ii) the variance sensitivity to variance of market variance, $\mathbb{C}\mathrm{ov}_t\left[V_{i,t+1}, Q_{m,t+1}\right]$. The first term corresponds to the variance beta of Carr and Wu (2009). The second term measures the risk that individual stock volatility co-moves with the market volatility of volatility. As shown by Carr and Wu (2009), variance risk premium corresponds to a trading strategy that shorts a swap on the realized variance; in particular, $\mathbb{E}_t^{\mathbb{Q}}\left[V_{i,t+1}\right]$ is the price for the contract and $\mathbb{E}_t\left[V_{i,t+1}\right]$ is the expected payoff. Selling a volatility asset with high the volatility sensitivity to market volatility-of-volatility requires high insurance payment since the asset can hedge away the upward market volatility-of-volatility during the market downturns.

Consistent with our model, by sorting stocks into quintile portfolios based on variance sensitivities to market volatility-of-volatility, we find that stock with high sensitivities have higher one-month variance risk premium than stocks with low sensitivities by 67.7 (in percentages squared) per month. The magnitude of the cross-sectional difference in variance risk premium is large compared to the market variance risk premium, which is 17.3 (in percentages squared) per month during our sample period. We study how volatility-of-volatility affects the variance risk premium by running the cross-sectional regressions on the 25 testing portfolios formed on the variance sensitivities to market volatility-of-volatility. We find that the risk price of variance beta with respect to variance of market variance is significantly positive. These findings suggest that market volatility-of-volatility is a priced factor in the cross-sectional variance risk premium.

Our study could also be motivated by the recent finding in Bollerslev, Tauchen, and Zhou (2009) that the variance risk premium of aggregate stock market returns has outstanding predictive power for future aggregate stock market return. The underlying mechanism in their work is that the state variable, the variance of economic variance, which affects expected market returns and solely determines the variance risk premium, delivers the predictability. Their work motivates several papers to focus on various economic mechanisms behind the return predictability afforded by variance risk premium. For example, Drechsler and Yaron (2011) show that jump shocks, in a more elaborate LRR model, capture the size and predictive power of the variance premium. Moreover, Drechsler (2013) show that model uncertainty has a large impact on variance risk premium, helping explain its power to predict stock returns. Nevertheless, none of prior studies provides evidence that volatility-of-volatility is a priced risk factor important for cross-sectional asset pricing.

Our paper is related to the pricing model with higher moments of the market return as risk factors studied by Chang, Christoffersen, and Jacobs (2013). They find that market skewness is a priced risk factor in the cross section of stock returns. Both our paper and their work extend the investigation of Ang, Hodrick, Xing, and Zhang (2006) and extract implied moments from index option prices. However, our results are robust to the inclusion of market skewness factor while the market skewness risk premium is much weaker in our sample period when we control for our market volatility-of-volatility risk.

Our paper is also related to but different from Han and Zhou (2012). They examine how firm-level variance risk premiums affect the stock returns in the cross-section, but they do not develop any theory to explain the dependencies. In contrast, our study investigates specifically the pricing of variance of market variance in the joint of cross-sectional stock returns and variance risk premium.

Finally, independent to our study, Baltussen, Van Bekkum, and Van Der Grient (2013) develop a measure of ambiguity, based on firm-level historical volatility of individual option-implied volatility (vol-of-vol). They find that vol-of-vol affects expected stock returns but their results cannot confirm that vol-of-vol is a priced risk factor. Our investigation differs with theirs in two aspects. First, our measure is based on intraday variation of market variance, resulting in a market volatility-of-volatility factor of daily frequency, while their vol-of-vol is based on historical daily information of implied volatility, resulting in a firm-level uncertainty measure of monthly frequency. Second, we find evidence for the rational pricing of market volatility-of-volatility risk, which sharply contrasts their ambiguity interpretation.

The remainder of the paper is organized as follows. The next section describes the economic dynamics and develops our market-based three-factor model for the empirical implementation. Section 3 constructs the measure of market volatility-of-volatility. Section 4 describes the data and presents the summary statistics. In section 5, we show empirical evidence on the pricing of variance of market variance risk in cross-sectional stock returns. Section 6 provides evidence in cross-sectional variance risk premium. The return predictability for the aggregate market portfolio is examined in section 7. Finally, section 8 contains our concluding remarks.

2. A three-factor model

This section describes the economic model. Our model begins with a macroeconomic model that incorporates the seminal long-run risks (LRR) model of Bansal and Yaron (2004) and the variance-of-variance model of Bollerslev, Tauchen, and Zhou (2009). We solve the macro-finance model explicitly and derive the equilibrium aggregate asset prices. Then, we use the properties of aggregate asset prices to characterize the macroeconomic risks and develop a market-based three-factor model for the cross-sectional asset prices.

2.1. Economic dynamics and equilibrium aggregate asset prices

The underlying economy is a discrete time endowment economy. The dynamics of consumption growth rate, g_{t+1}, and dividend growth rate, $g_{d,t+1}$, are governed by the following process:

$$
\begin{aligned}
g_{t+1} &= \mu_g + x_t + \sigma_t z_{g,t+1} \\
x_{t+1} &= \rho_x x_t + \varphi_x \sigma_t z_{x,t+1} \\
\sigma_{t+1}^2 &= \mu_\sigma + \rho_\sigma \sigma_t^2 + q_t z_{\sigma,t+1} \\
q_{t+1}^2 &= \mu_q + \rho_q q_t^2 + \varphi_q z_{q,t+1} \\
g_{d,t+1} &= \mu_d + \phi x_t + \varphi_d \sigma_t z_{d,t+1}
\end{aligned}
\tag{1}
$$

$$z_{g,t+1}, z_{x,t+1}, z_{\sigma,t+1}, z_{q,t+1}, z_{d,t+1} \overset{\text{iid}}{\sim} N(0,1)$$

where x_{t+1} represents the long-run consumption growth, σ_{t+1}^2 is the time-varying economic uncertainty, and $q_{t+1}^2$is the economic volatility-of-volatility, which is the conditional variance of the economic uncertainty. The features of the long-run risk and the time-varying economic uncertainty is proposed by Bansal and Yaron (2004), while the additional feature of economic volatility-of-volatility is introduced by Bollerslev, Tauchen, and Zhou (2009). The representative agent is equipped with recursive preferences of Epstein and Zin (1989). Thus, the logarithm of the Intertemporal Marginal Rate of Substitution (IMRS), m_{t+1}, is

$$m_{t+1} = \theta \log(\delta) - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{a,t+1}, \tag{2}$$

where $r_{a,t+1}$ is the return on consumption claim, and $\theta \equiv (1-\gamma)(1-1/\psi)^{-1}$. We assume that $\gamma > 1$, and $\psi > 1$, and therefore $\theta < 0$. Based on Campbell and Shiller (1988) approximation, $r_{a,t+1} \approx \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1}$, where z_t is the logarithm of price–consumption ratio, which in equilibrium is an affine function of the state variables, $z_t = A_0 + A_x x_t + A_\sigma \sigma_t^2 + A_q q_t^2$.[5]

Substituting the equilibrium consumption return, $r_{a,t+1}$, into the IMRS, the innovation in the pricing kernel m_{t+1} is

$$\begin{aligned} m_{t+1} - \mathbb{E}_t[m_{t+1}] &= -\lambda_g \sigma_t z_{g,t+1} \\ &\quad - \lambda_x \sigma_t z_{x,t+1} - \lambda_\sigma q_t z_{\sigma,t+1} - \lambda_q \varphi_q z_{q,t+1} \end{aligned} \tag{3}$$

where $\lambda_g = \gamma > 0$, $\lambda_x = (1-\theta) A_x \kappa_1 \varphi_x > 0$, $\lambda_\sigma = (1-\theta) A_\sigma \kappa_1 < 0$, $\lambda_q = (1-\theta) A_q \kappa_1 < 0$. The parameters determine the prices for short-run risk (λ_g), long-run risk (λ_x), volatility risk (λ_σ), and volatility of volatility risk (λ_q).

An analogous expression holds for the stock market return, $r_{m,t+1} = \kappa_{0,m} + \kappa_{1,m} z_{m,t+1} - z_{m,t} + g_{d,t+1}$, where $z_{m,t}$ is the log price–dividend ratio, which in equilibrium is an affine function of the state variables, $z_{m,t} = A_{0,m} + A_{x,m} x_t + A_{\sigma,m} \sigma_t^2 + A_{q,m} q_t^2$.[6] Since we require that $\theta < 0$, we have $A_{x,m} > 0$, $A_{\sigma,m} < 0$, and $A_{q,m} < 0$. The innovation in market return can be express as

$$\begin{aligned} r_{m,t+1} - \mathbb{E}_t\left[r_{m,t+1}\right] &= \varphi_d \sigma_t z_{d,t+1} \\ &\quad + \beta_{m,x} \sigma_t z_{x,t+1} + \beta_{m,\sigma} q_t z_{\sigma,t+1} + \beta_{m,q} \varphi_q z_{q,t+1}, \end{aligned} \tag{4}$$

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[5] The equilibrium solutions for the coefficients are:

$A_x = \frac{1-1/\psi}{1-\kappa_1 \rho_x} > 0, A_\sigma = \frac{\theta((1-1/\psi)^2 + A_x^2 \kappa_1^2 \varphi_x^2)}{2(1-\kappa_1 \rho_\sigma)} < 0,$ and $A_q = \frac{\theta A_\sigma^2 \kappa_1^2}{2(1-\kappa_1 \rho_q)} < 0.$

[6] The equilibrium solutions for the coefficients are: $A_{x,m} = \frac{\phi - 1/\psi}{1-\kappa_{1,m}\rho_x}$, $A_{\sigma,m} = \frac{(1-\theta)A_\sigma(1-\kappa_1\rho_\sigma) + 0.5 H_{m,\sigma}}{1-\kappa_{1,m}\rho_\sigma}$, and $A_{q,m} = \frac{(1-\theta)A_q(1-\kappa_1\rho_q) + 0.5 H_{m,q}}{1-\kappa_{1,m}\rho_q}$, where $H_{m,\sigma} = \gamma^2 + \varphi_d^2 + \varphi_x^2(\lambda_x - \beta_{m,x})^2$ and $H_{m,q} = (\lambda_\sigma - \beta_{m,\sigma})^2$.

where $\beta_{m,x} = A_{x,m}\kappa_{1,m}\varphi_x > 0$, $\beta_{m,\sigma} = A_{\sigma,m}\kappa_{1,m} < 0$, $\beta_{m,q} = A_{q,m}\kappa_{1,m} < 0$. It is straightforward now to derive the equity premium on the market portfolio,

$$\begin{aligned} \mathbb{E}_t[r_{m,t+1}] - r_{f,t} + 0.5\mathbb{V}\mathrm{ar}_t[r_{m,t+1}] &= \mathbb{C}\mathrm{ov}_t[r_{m,t+1}, -m_{t+1}] \\ &= \lambda_x\beta_{m,x}\sigma_t^2 + \lambda_\sigma\beta_{m,\sigma}q_t^2 + \lambda_q\beta_{m,q}\varphi_q^2. \end{aligned} \tag{5}$$

The expected market return consists of three terms. The first two terms are long-run risk premium and volatility risk premium, which are the same as in Bansal and Yaron (2004), while the last term represents the volatility-of-volatility risk premium, which corresponds to the work of Bollerslev, Tauchen, and Zhou (2009).[7]

The conditional variance of market return is readily calculated as $V_{m,t} \equiv \mathbb{V}\mathrm{ar}_t[r_{m,t+1}] = (\varphi_d^2 + \beta_{m,x}^2)\sigma_t^2 + \beta_{m,\sigma}^2 q_t^2 + \beta_{m,q}^2\varphi_q^2$, and the process for innovations in market variance is

$$V_{m,t+1} - \mathbb{E}_t[V_{m,t+1}] = \beta_{V,\sigma}q_t z_{\sigma,t+1} + \beta_{V,q}\varphi_q z_{q,t+1}, \tag{6}$$

where $\beta_{V,\sigma} = \varphi_d^2 + \beta_{m,x}^2$, $\beta_{V,q} = \beta_{m,\sigma}^2$. Thus, innovations in market variance are related to both the economic volatility shock and the economic volatility-of-volatility shock. It follows that the market volatility-of-volatility (e.g. the conditional variance of market variance) is $Q_{m,t} \equiv \mathbb{V}\mathrm{ar}_t[V_{m,t+1}] = \beta_{V,\sigma}^2 q_t^2 + \beta_{V,q}^2\varphi_q^2$, and the process for its innovations is

$$Q_{m,t+1} - \mathbb{E}_t[Q_{m,t+1}] = \beta_{Q,q}\varphi_q z_{q,t+1}, \tag{7}$$

where $\beta_{Q,q} = \beta_{V,\sigma}^2$. Note that innovations in market volatility-of-volatility is solely determined by economic variance of variance shock with a scaling factor, $\beta_{Q,q}$. The market volatility-of-volatility-of-volatility (e.g. the conditional variance of variance of market variance), $W_{m,t} \equiv \mathbb{V}\mathrm{ar}_t[Q_{m,t+1}] = \beta_{Q,q}^2\varphi_q^2$, is constant in our model.

Next, we consider the market variance risk premium, which is defined as the

[7] Since we do not assume the square root process for the volatility-of-volatility as Bollerslev, Tauchen, and Zhou (2009) do, the volatility risk in the resulting equity premium does not confound with the volatility-of-volatility risk.

difference between the conditional variance under risk-neutral measure and the conditional variance under physical measure. Under the risk-neutral measure, which is characterized by the Radon-Nikodym derivative $\frac{\mathrm{d}\mathbb{Q}}{d\mathbb{P}} = \frac{\exp(m_{t+1})}{\mathbb{E}_t[\exp(m_{t+1})]}$, the economy dynamics preserve the same structure but with a shift the mean.[8] Therefore, our model implies that both the conditional variance of market return and the conditional variance of market variance are invariant under the risk neutral measure; that is, $V_{m,t}^{\mathbb{Q}} \equiv \mathbb{V}\mathrm{ar}_t^{\mathbb{Q}}\left[r_{m,t+1}\right] = V_{m,t}$, and $Q_{m,t}^{\mathbb{Q}} \equiv \mathbb{V}\mathrm{ar}_t^{\mathbb{Q}}\left[V_{m,t+1}\right] = Q_{m,t}$. The market variance risk premium can be expressed as

$$\begin{aligned} VRP_{m,t} &\equiv \mathbb{E}_t^{\mathbb{Q}}\left[V_{m,t+1}\right] - \mathbb{E}_t\left[V_{m,t+1}\right] \\ &= \beta_{V,\sigma} q_t \left(\mathbb{E}_t^{\mathbb{Q}}\left[z_{\sigma,t+1}\right] - \mathbb{E}_t\left[z_{\sigma,t+1}\right]\right) \\ &\qquad + \beta_{V,q} \varphi_q \left(\mathbb{E}_t^{\mathbb{Q}}\left[z_{q,t+1}\right] - \mathbb{E}_t\left[z_{q,t+1}\right]\right) \\ &= -\lambda_\sigma \beta_{V,\sigma} q_t^2 - \lambda_q \beta_{V,q} \varphi_q^2. \end{aligned} \tag{8}$$

The negative volatility risk price (λ_σ) and the negative volatility-of-volatility risk price (λ_q) contributes to the positive market variance risk premium. Moreover, the model predicts that the market variance risk premium is time-varying and is entirely driven by the dynamics of the economic volatility-of-volatility (q_t^2), which corresponds to the work of Bollerslev, Tauchen, and Zhou (2009).

2.2. Leverage effects, feedback effects, and return predictability

The model endogenously generates an asymmetric return-volatility dependency. In the literature, leverage effect (e.g. Black, 1976; Christie, 1982; among others) refers to the negative contemporaneous return-volatility correlation, while the

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[8] That is,

$$\begin{aligned} g_{t+1} &= \left(\mu_g - \gamma\sigma_t^2\right) + x_t + \sigma_t z_{g,t+1}^{\mathbb{Q}}, \\ x_{t+1} &= -\lambda_x \sigma_t^2 + \rho_x x_t + \varphi_x \sigma_t z_{x,t+1}^{\mathbb{Q}}, \\ \sigma_{t+1}^2 &= \left(\mu_\sigma - \lambda_\sigma q_t^2\right) + \rho_\sigma \sigma_t^2 + q_t z_{\sigma,t+1}^{\mathbb{Q}}, \\ q_{t+1}^2 &= \left(\mu_q - \lambda_q \varphi_q^2\right) + \rho_q q_t^2 + \varphi_q z_{q,t+1}^{\mathbb{Q}}, \\ g_{d,t+1} &= \mu_d + \phi x_t + \varphi_d \sigma_t z_{d,t+1}^{\mathbb{Q}}, \end{aligned}$$

where $z_{g,t+1}^{\mathbb{Q}} = \gamma\sigma_t + z_{g,t+1}$, $z_{x,t+1}^{\mathbb{Q}} = \lambda_x\sigma_t + z_{x,t+1}$, $z_{\sigma,t+1}^{\mathbb{Q}} = \lambda_\sigma q_t + z_{\sigma,t+1}$, $z_{q,t+1}^{\mathbb{Q}} = \lambda_q\varphi_q + z_{q,t+1}$, and $z_{d,t+1}^{\mathbb{Q}} = z_{d,t+1}$.

mechanism of volatility feedback effect (see, e.g. Campbell and Hentschel 1992; Bekaert and Wu 2000; Wu 2001; Bollerslev, Sizova, and Tauchen, 2012; among others) is often used to explain the positive correlations between future returns and volatility. Our model is in line with the leverage effect and the feedback effect; that is, a straightforward calculation shows that

$$\begin{aligned}
\mathbb{C}\mathrm{ov}_t\left[r_{m,t+1}, V_{m,t+1}\right] &= \beta_{m,\sigma}\beta_{V,\sigma}q_t^2 + \beta_{m,q}\beta_{V,q}\varphi_q^2 < 0, \\
\mathbb{C}ov_t\left[r_{m,t+1+j}, V_{m,t+1}\right] &= \mathbb{C}ov_t\left[\mathbb{E}_{t+1}\left[\dots\ \mathbb{E}_{t+j}\left[r_{m,t+1+j}\right]\right], V_{m,t+1}\right] \\
&= -\kappa_\sigma\, \rho_\sigma^j \beta_{m,\sigma}\beta_{V,\sigma}q_t^2 - \kappa_q \rho_q^j\, \beta_{m,q}\beta_{V,q}\varphi_q^2 > 0
\end{aligned} \tag{9}$$

where $\kappa_\sigma = (1-\kappa_{1,m}\rho_\sigma)/\kappa_{1,m}$ and $\kappa_q = (1-\kappa_{1,m}\rho_q)/\kappa_{1,m}$. In the absence of the time-varying economic volatility-of-volatility (e.g. when q_t^2 is constant and φ_q^2=0), the second term of $\mathbb{C}\mathrm{ov}_t\left[r_{m,t+1}, V_{m,t+1}\right]$ and the second term of $\mathbb{C}ov_t\left[r_{m,t+1+j}, V_{m,t+1}\right]$ are reduced to zero, leading both of the two covariances to smaller values. Thus, the dynamics of economic volatility-of-volatility amplifies the leverage effect and the volatility feedback effect.

Moreover, our model implies the existence of leverage effect and feedback effect related to market volatility-of-volatility. The contemporaneous and forward correlations between market return and market volatility-of-volatility can be expressed as

$$\begin{aligned}
\mathbb{C}\mathrm{ov}_t\left[r_{m,t+1}, Q_{m,t+1}\right] &= \beta_{m,q}\beta_{Q,q}\varphi_q^2 < 0, \\
\mathbb{C}ov_t\left[r_{m,t+1+j}, Q_{m,t+1}\right] &= -\kappa_q\rho_q^j\, \beta_{m,q}\beta_{Q,q}\varphi_q^2 > 0.
\end{aligned} \tag{10}$$

If volatility-of-volatility is priced, an anticipated increase in volatility-of-volatility raises the required rate of return, implying an immediate stock price decline and higher future returns. Thus, the above expressions provide important and directly testable implications for the volatility-of-volatility risk premium.

It is instructive to consider the return predictability afforded by the market variance risk premium, which is the main proposition in the pioneer work of Bollerslev, Tauchen, and Zhou (2009). In our model, the process for innovations in the market variance risk premium is

$$VRP_{m,t+1} - \mathbb{E}_t\left[VRP_{m,t+1}\right] = -\lambda_\sigma \beta_{V,\sigma} \varphi_q z_{q,t+1}, \tag{11}$$

which is entirely determined by economic volatility-of-volatility shock like the market volatility-of-volatility is. Thus, similar to Bollerslev, Tauchen, and Zhou (2009), market variance risk premium can predict the future market return as follows,

$$\mathbb{C}ov_t\left[r_{m,t+1+j}, VRP_{m,t+1}\right] = \kappa_q \rho_q^j \, \beta_{m,q} \lambda_\sigma \beta_{V,\sigma} \varphi_q^2 > 0. \tag{12}$$

In the presence of jumps, however, as indicated in Drechsler and Yaron (2011), the market variance risk premium is affected by risk of jumps that is also likely to deliver the return predictability. In which case, the market variance risk premium is no longer solely driven by the economic volatility-of-volatility, lowering the testing power of Bollerslev, Tauchen, and Zhou (2009) for the return predictability afforded by market variance risk premium against an alternative source of risk. Nevertheless, the continuous component of the market volatility-of-volatility is still corresponding to the economic volatility-of-volatility. Thus, for the testing power consideration, the empirical strategy of our model focuses on the identification of the continuous component of the market volatility-of-volatility.

2.3. A market-based three-factor model for individual stocks

We assume that the innovations in stock return *i* is

$$r_{i,t+1} - \mathbb{E}_t\left[r_{i,t+1}\right] = \beta_{i,x} \sigma_t z_{x,t+1} + \beta_{i,\sigma} q_t z_{\sigma,t+1} + \beta_{i,q} \varphi_q z_{q,t+1}. \tag{13}$$

Given the expression for the pricing kernel in equation(3), the expected stock return can be written as

$$\mathbb{E}_t[r_{i,t+1}] - r_{f,t} + 0.5\mathbb{V}\text{ar}_t[r_{i,t+1}] \\ = \beta_{i,x}\lambda_x\sigma_t^2 + \beta_{i,\sigma}\lambda_\sigma q_t^2 + \beta_{i,q}\lambda_q\varphi_q^2. \tag{14}$$

Thus, the expected stock return is determined by three sources of economic risks: economic long-run risk ($\beta_{i,x}$), economic volatility risk ($\beta_{i,\sigma}$), and economic volatility-of- volatility risk ($\beta_{i,q}$).

We now use the properties of the aggregate asset prices to characterize the macroeconomic risks. First of all, in equilibrium, the market volatility-of-volatility risk, which is the return covariance with respect to variance of market variance, is solely determined by the economic volatility-of-volatility risk ($\beta_{i,q}$), i.e.

$$\mathbb{C}\text{ov}_t[r_{i,t+1}, Q_{m,t+1}] = \beta_{i,q}\beta_{Q,q}\varphi_q^2. \tag{15}$$

Furthermore, the return sensitivities with respect to market variance and with respect to market return provide additional information for the economic volatility risk and the long-run risk; that is,

$$\mathbb{C}\text{ov}_t[r_{i,t+1}, V_{m,t+1}] = \beta_{i,\sigma}\beta_{V,\sigma}q_t^2 + \beta_{i,q}\beta_{V,q}\varphi_q^2, \tag{16}$$

$$\mathbb{C}\text{ov}_t[r_{i,t+1}, r_{m,t+1}] = \beta_{i,x}\beta_{m,x}\sigma_t^2 + \beta_{i,\sigma}\beta_{m,\sigma}q_t^2 + \beta_{i,q}\beta_{m,q}\varphi_q^2. \tag{17}$$

Substituting out the economic risks in (14) with (15), (16) and (17) gives us the market-based three-factor model:

$$\mathbb{E}_t[r_{i,t+1}] - r_{f,t} + 0.5\mathbb{V}\text{ar}_t[r_{i,t+1}] \\ = \lambda_m\mathbb{C}\text{ov}_t[r_{i,t+1}, r_{m,t+1}] + \lambda_V\mathbb{C}\text{ov}_t[r_{i,t+1}, V_{m,t+1}] \\ + \lambda_Q\mathbb{C}\text{ov}_t[r_{i,t+1}, Q_{m,t+1}], \tag{18}$$

where $$\lambda_m = \frac{\lambda_x}{\beta_{m,x}}, \lambda_V = \frac{\lambda_\sigma - \lambda_m\beta_{m,\sigma}}{\beta_{V,\sigma}}, \lambda_Q = \frac{\lambda_q - \lambda_V\beta_{V,q} - \lambda_m\beta_{m,q}}{\beta_{Q,q}}. \tag{19}$$

Thus, the expected stock return is now determined by three sources of risks related to aggregate asset prices. The first term measures the market risk of classical capital asset pricing model (CAPM; Sharpe, 1964; Lintner, 1965). The second term corresponds to the aggregate volatility risk of Ang, Hodrick, Xing, and Zhang (2006). The last term, which is the main focus of this paper, measures the aggregate volatility of volatility risk. The resulting three risk prices in our market-based model, λ_m, λ_V, and λ_Q, are related to the three economic risk prices with a linear transformation.

The market-based model developed in this paper has several advantages. First, financial data provide useful information because asset prices tell us how market participants value risks. Moreover, financial data convey information to public in a timely fashion. Hence, the empirical design of our model is compatible with a large literature of multi-factor model explaining cross-sectional monthly stock returns (see, for instance, Fama and French, 1993; Ang, Hodrick, Xing, and Zhang, 2006; Maio and Santa-Clara, 2012; among others).

It is constructive to establish the individual variance risk premiums under the proposed model. The conditional variance of the time t to $t+1$ return of stock i ($r_{i,t+1}$) and the innovations in conditional variance i can be expressed as

$$\begin{aligned} V_{i,t} &\equiv \mathbb{V}\mathrm{ar}_t\left[r_{i,t+1}\right] = \beta_{i,x}^2\sigma_t^2 + \beta_{i,\sigma}^2 q_t^2 + \beta_{i,q}^2\varphi_q^2 \\ V_{i,t+1} - \mathbb{E}_t\left[V_{i,t+1}\right] &= \beta_{i,\sigma}^V q_t z_{\sigma,t+1} + \beta_{i,q}^V \varphi_q z_{q,t+1} \end{aligned} \tag{20}$$

where $\beta_{i,\sigma}^V = \beta_{i,x}^2$ and $\beta_{i,q}^V = \beta_{i,\sigma}^2$. It follows that

$$VRP_{i,t} \equiv \mathbb{E}_t^{\mathbb{Q}}\left[V_{i,t+1}\right] - \mathbb{E}_t\left[V_{i,t+1}\right] = -\lambda_\sigma\beta_{i,\sigma}^V q_t^2 - \lambda_q\beta_{i,q}^V\varphi_q^2, \tag{21}$$

which suggests that the individual variance risk premiums are determined by the conditional variance's betas with respect to the economic volatility risk ($\beta_{i,\sigma}^V$) and with respect to the economic volatility-of-volatility risk ($\beta_{i,q}^V$).

To derive a market-based variance risk premium model, we consider the variance sensitivities with respect to market variance and with respect to market volatility-of-volatility, which in the equilibrium are given by

$$\mathbb{C}\mathrm{ov}_t\left[V_{i,t+1}, Q_{m,t+1}\right] = \beta_{i,q}^V \beta_{Q,q} \varphi_q^2, \tag{22}$$

$$\mathbb{C}\mathrm{ov}_t\left[V_{i,t+1}, V_{m,t+1}\right] = \beta_{i,\sigma}^V \beta_{V,\sigma} q_t^2 + \beta_{i,q}^V \beta_{V,q} \varphi_q^2. \tag{23}$$

Similarly, substituting out the economic risks in (21) with (22) and (23) gives us the market-based two-factor model for the individual variance risk premium:

$$VRP_{i,t} = -\lambda_V^V \mathbb{C}\mathrm{ov}_t\left[V_{i,t+1}, V_{m,t+1}\right] - \lambda_Q^V \mathbb{C}\mathrm{ov}_t\left[V_{i,t+1}, Q_{m,t+1}\right], \tag{24}$$

where
$$\lambda_V^V = \frac{\lambda_\sigma}{\beta_{V,\sigma}} \text{ and } \lambda_Q^V = \frac{\lambda_q - \lambda_V^V \beta_{V,q}}{\beta_{Q,q}}. \tag{25}$$

Therefore, the individual variance risk premium is now determined by two sources of risks related to aggregate asset prices. The first term corresponds to the variance beta of Carr and Wu (2009). The second term measures the risk that individual stock volatility co-moves with the aggregate volatility of volatility. The resulting two risk prices associated with the individual variance risk premium, λ_V^V and λ_Q^V, are also related to the corresponding economic risk prices, λ_σ and λ_q, with a linear transformation.

Our model implies that the three aggregate asset prices are inter-dependent and so are the market-based risks in the individual expected return and variance risk premium. Moreover, the risk prices for the high moments are offset by the risk prices for the low moments. While this property is interesting, it also complicates the task of distinguishing the relative impacts from the underlying sources of risks. Nevertheless, our market-based model can be alternatively implemented using orthogonalized aggregate asset prices as risk factors. Define $\tilde{Q}_{m,t+1} = Q_{m,t+1}, \tilde{V}_{m,t+1} = V_{m,t+1} - \mathbb{E}\left[V_{m,t+1} | Q_{m,t+1}\right]$, and $\tilde{r}_{m,t+1} = r_{m,t+1} - \mathbb{E}\left[r_{m,t+1} | V_{m,t+1}, Q_{m,t+1}\right]$. Thus, each of the market-based risks is directly linked to the counterpart of the underlying economic risks. In which case, the expected stock return is represented by $\tilde{\lambda}_m \mathbb{C}\mathrm{ov}_t\left[r_{i,t+1}, \tilde{r}_{m,t+1}\right] + \tilde{\lambda}_V \mathbb{C}\mathrm{ov}_t\left[r_{i,t+1}, \tilde{V}_{m,t+1}\right] + \tilde{\lambda}_Q \mathbb{C}\mathrm{ov}_t\left[r_{i,t+1}, \tilde{Q}_{m,t+1}\right]$ and the individual variance risk premium can also be expressed by $\tilde{\lambda}_V \mathbb{C}\text{ov}_t\left[V_{i,t+1}, \tilde{V}_{m,t+1}\right] + \tilde{\lambda}_Q \mathbb{C}\text{ov}_t\left[V_{i,t+1}, \tilde{Q}_{m,t+1}\right]$. Thus, the resulting risk prices preserve the sign of the original economic risk prices; that is, $\tilde{\lambda}_m = \lambda_x/\beta_{m,x}$, $\tilde{\lambda}_V = \lambda_\sigma/\beta_{V,\sigma}$, $\tilde{\lambda}_Q = \lambda_q/\beta_{Q,q}$.

3. Estimation of variance of market variance

In previous section, we propose a market-based three-factor model, which requires the information of market return, market variance, and variance of market variance. To proxy for the first two factors, we use CRSP value-weighted market index and CBOE VIX index, which have been widely used in the literature (see, for example, Ang, Hodrick, Xing, and Zhang, 2006; Chang, Christoffersen, and Jacobs, 2013; Bollerslev, Tauchen, and Zhou, 2009; among others). In this study, we estimate the variance of market variance by calculating the realized bipower variation from a series of five-minute model-free implied variances, using the high-frequency S&P 500 index option data. The details of our empirical settings are described as follows.

First of all, we extract the model-free implied variance, using the spanning methodology proposed by Carr and Madan (2001), Bakshi and Madan (2000), Bakshi, Kapadia, and Madan (2003), and Jiang and Tian (2005). Bakshi, Kapadia, and Madan (2003) show that the price of a τ-maturity return variance contract, which is the discounted conditional expectation of the square of market return under the risk-neutral measure, can be spanned by a collection of out-of-money call options and out-of-money put options,

$$\begin{aligned} \bar{V}_t(\tau) &\equiv \mathbb{E}_t^{\mathbb{Q}}\left[\mathrm{e}^{-r_{f,t}}\mathrm{Log}\left[\frac{S_{t+\tau}}{S_t}\right]^2\right] \\ &= \int_{S_t}^{\infty} \frac{2(1-\log[K/S_t])}{K^2} C_t(K;\tau)\, dK \\ &\quad + \int_0^{S_t} \frac{2(1+\log[K/S_t])}{K^2} P_t(K;\tau)\, dK, \end{aligned} \tag{26}$$

where $C_t(K;\tau)$ and $P_t(K;\tau)$ are the prices of European calls and puts at time t written on the underlying stock with strike price K and expiration date at $t+\tau$. The conditional variance of market return can be calculated by

$$IV_t(\tau) = \mathrm{e}^{r_{f,t}}\bar{V}_t(\tau) - \mu_t(\tau)^2, \tag{27}$$

where $\mu_t(\tau)$ satisfies the risk-neutral valuation relationship, which is related to the first four risk-neutral moments of market returns as described in equation (39) of Bakshi, Kapadia, and Madan (2003).

Second, we use the model-free realized bipower variance, introduced by Bardorff-Nielsen and Shephard (2004), to estimate the variance of market variance. Define the intraday stock return as $r_{t+1,j} \equiv \log[S_{t+j/M}] - \log[S_{t+(j-1)/M}]$, $j = 1, \dots, M$, where M is the sampling frequency per trading day. Bardorff-Nielsen and Shephard (2004) study two measures of realized variations; the first one is the realized variation, RV_{t+1}, and the second one is the bipower variation, BV_{t+1}:

$$RV_{t+1} = \sum_{j=1}^{M} r_{t+1,j}^2, \tag{28}$$

$$BV_{t+1} = \frac{\pi}{2}\left(\frac{M}{M-1}\right)\sum_{j=2}^{M} |r_{t+1,j}||r_{t+1,j-1}|. \tag{29}$$

Andersen, Bollerslev, and Diebold (2002) show that the realized variance converges to the integrated variance plus the jump contributions, i.e.

$$RV_{t+1} \xrightarrow{M\to\infty} \int_t^{t+1} \sigma^2(s)\,ds + \sum_{j=1}^{N_{t+1}} J_{t+1,j}^2, \tag{30}$$

where N_{t+1} is the number of return jumps within day t+1 and $J_{t+1,j}^2$ is the jump size. Moreover, Bardorff-Nielsen and Shephard (2004) show that

$$BV_{t+1} \xrightarrow{M\to\infty} \int_t^{t+1} \sigma^2(s)\,ds. \tag{31}$$

In other words, bipower variation provides a consistent estimator of the integrated variance solely for the diffusion part.

Our measure for variance of market variance is estimated from a series of five-minute based model-free implied variances. The intraday model-free implied variances are calculated using equation (27), which is denoted as $IV_{t+j/M}(\tau), j = 1, \ldots, M$. Since the process of market variance is a (semi-)martingale, we apply the bipower variation formula on the changes in annualized model-free implied variances and obtain a measure for variance of market variance:

$$VoV_{t+1}(\tau) = \frac{\pi}{2}\left(\frac{M}{M-1}\right)\sum_{j=2}^{M}\left|\Delta v_{t+1,j}(\tau)\right|\left|\Delta v_{t+1,j-1}(\tau)\right| \tag{32}$$

where $\Delta v_{t+1,j}(\tau) \equiv \frac{365}{\tau}\left[IV_{t+j/M}(\tau) - IV_{t+(j-1)/M}(\tau)\right]$. In this way, our empirical results will not be affected by the volatility jumps (or the return jumps embedded in the volatility).

4. Data and descriptive statistics

4.1. Data description

We use the tick-by-tick quoted data on S&P 500 index (SPX) options from CBOE's Market Data Report (MDR) tapes over the time period from January 1996 to December 2010. The underlying SPX prices are also provided in the tapes. We obtain daily data from OptionMetrics for equity options and S&P 500 index options. We use the Zero Curve file, which contains the current zero-coupon interest rate curve, and the Index Dividend file, which contains the current dividend yield, from OptionMetrics to calculate the implied volatility for each tick-by-tick data from CBOE's MDR tapes. Daily and monthly stock return data are from CRSP while intraday transactions data are from TAQ data sets. Financial statement data are from COMPUSTAT. Fama and French (1993) factors and their momentum *UMD* factor are obtained from the online data library of Ken French.[9] *VIX* index is obtained from the

[9] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/

website of CBOE.[10] While we use the 'new' *VIX* index to calculate the market variance risk premium as proposed by Bollerslev, Tauchen, and Zhou (2009), we also use the 'old' *VIX*, which is based on the S&P 100 options and Black–Scholes implied volatilities, as our volatility factor, following Ang, Hodrick, Xing, and Zhang (2006). We use the index option prices from the Option Price file to replicate the market skewness factor and the market kurtosis factor of Chang, Christoffersen, and Jacobs (2013).

We follow the literature (see, for example, Jiang and Tian 2005; Chang, Christoffersen, and Jacobs, 2013; among others) to filter out index option prices that violate the arbitrage bounds.[11] We also eliminate in-the-money options (e.g. put options with K/S>1.03 and call options with K/S<1.03) because prior study suggests that they are less liquid. We use the daily SPX low and high prices, downloaded from Yahoo Finance,[12] to filter out the MDR data that are outside the [low, high] interval.

For the computation of the market volatility-of-volatility, we first partition the tick-by-tick S&P500 index options data into five-minute intervals. For each maturity within each interval, we linearly interpolate implied volatilities for a fine grid of one thousand moneyness levels (K/S) between 0.01% and 300%[13] and use equations (26) and (27) to estimate the model-free implied variance. We then use linearly interpolate maturities to obtain the estimate at a fixed 30-day horizon. For each day, our measure for market volatility-of-volatility (*VoV*) is calculated by using the bipower variation formula of equation (32) with the 81 within-day five-minute annualized 30-day

[10] http://www.cboe.com/micro/vix/historical.aspx
[11] Moreover, we eliminate all observations for which the ask price is lower than the bid price, the bid price is equal to zero, or the average of the bid and ask price is less than 3/8.
[12] http://finance.yahoo.com/q/hp?s=^GSPC+Historical+Prices
[13] For moneyness levels below or above the available moneyness level in the market, we use the implied volatility of the lowest or highest available strike price.

model-free implied variance estimates covering the normal CBOE trading hours from 8:30 a.m. to 3:15 p.m. Central Time.

The market variance risk premium ($VRP_{m,t}$), following Bollerslev, Tauchen, and Zhou (2009), is defined as the difference between the ex-ante implied variance ($IV_{m,t}$) and the ex-post realized variance ($RV_{m,t}$), i.e. $VRP_{m,t} \equiv IV_{m,t} - RV_{m,t}$. We focus on a fixed maturity of 30 days. Market implied variance ($IV_{m,t}$) is measured by the squared 'new' VIX index divided by 12. Summation of SPX five-minute squared logarithmic returns are used to calculate the market realized variance ($RV_{m,t}$). With eighty five- minute intervals per trading day and the overnight return, we construct the daily market realized variance, using a rolling window of 22 trading days starting from the current day.

We construct the individual model-free implied variance ($IV_{i,t}$) using equity options data from the Volatility Surface file that provides Black-Scholes implied volatilities for options with standard maturities and delta levels. The individual implied variance is estimated by applying the same methodology that we use for the index options on the equity options data with 30-day maturity.

To compute the individual realized variance ($RV_{i,t}$), we extract from TAQ database the intraday transaction and quote data within the normal trading hours from 9:30 a.m. to 4:00 p.m. Eastern Time. We first adopt the step-by-step cleaning procedures proposed by Bardorff-Nielsen, Hansen, Lunde, and Shephard (2009) to screen the TAQ high frequency data,[14] and then we follow Sadka (2006) to remove quotes in which the quoted spread is more than 25% and remove trades in which the absolute value of one-tick return is more than 25%. The resulting 78 five-minute trades and quotes per trading day in a rolling window of 22 trading days are

[14] We apply the rules of P1, P2, P3, Q1, Q2, T1, T2, and T3 as described in the section 3.1 of Bardorff-Nielsen, Hansen, Lunde, and Shephard (2009) to carry out the cleaning procedures.

separately used to calculate the trade-based daily individual realized variance ($RV_{i,t}^{T}$) and the quote-based daily individual realized variance ($RV_{i,t}^{Q}$). To avoid the effect from stale prices in trades or in quotes, we further require that the both the number of five-minute trades and that of quotes in the 22-day rolling window should be more than 78×11=858. To conserve space, we will focus on the trade-based realized variance, i.e. $RV_{i,t} = RV_{i,t}^{T}$, while the results for the quote-based measure are available upon request.

We estimate the monthly expected individual variance risk premium ($EVRP_{i,t}$) through a forecast model. We adopt a linear forecast model, following Drechsler and Yaron (2011) and Han and Zhou (2012), to estimate the expected realized variance ($ERV_{i,t}$) with the lagged realized variance and the lagged model-free implied variance measured at the end of the month.[15] Thus, the expected individual variance risk premium is defined as $EVRP_{i,t} = IV_{i,t} - ERV_{i,t}$.

To implement our empirical model, we construct innovations in market moments. First, following Ang, Hodrick, Xing, and Zhang (2006), innovations in market volatility (ΔVIX) is measured by its first order difference, i.e. $\Delta VIX_{t+1} = VIX_{t+1} - VIX_t$. Chang, Christoffersen, and Jacobs (2013) indicate that taking the first difference is appropriate for *VIX*, whereas an ARMA(1,1) model is need to remove the autocorrelation in the their skewness and kurtosis factors. Following their approach, the innovations in market volatility-of-volatility (ΔVoV) is computed as the ARMA(1,1) model residuals of the market volatility-of-volatility.

4.2. Descriptive statistics

The daily measure of *VoV* is plotted in Figure 1. There are clear spikes on the graph—the Asian financial crisis in1997, the LTCM crisis in1998, September11, 2001,

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[15] Specifically, for stock *i*, we run the regression: $RV_{i,t+1} = \alpha + \beta_0 IV_{i,t} + \beta_1 RV_{i,t}$ and defined the fitted value as $ERV_{i,t}$, i.e. $ERV_{i,t} \equiv \widehat{RV}_{i,t+1} = \hat{\alpha} + \hat{\beta}_0 IV_{i,t} + \hat{\beta}_1 RV_{i,t}$.

the WorldCom and Enron bankruptcies in 2001 and 2002, subprime loan crisis in 2007, the recent financial crisis in 2008, and the flash crash in 2010.

Table 1 reports descriptive statistics for the daily factors used in this paper. In our sample, the mean of 30-day market variance risk premium (*VRP*) is 17.260 (in percentages squared), which is slightly smaller than 18.3 in Bollerslev, Tauchen, and Zhou's (2010) sample. The mean of *VoV* is 0.054%, which is much smaller than its standard deviation, 0.563%. The mean of *SKEW* is -1.663 and the mean of *KURT* is 9.313. Thus, the risk-neutral distribution of the market return is asymmetric and has fat tails.

Panel B reports the Spearman correlations between factors, including the excess market return (*MKT*), the Fama and French (1993) *SMB* and *HML* factors, the momentum factor (*UMD*), the changes in *VIX* (*ΔVIX*; Ang, Hodrick, Xing, and Zhang, 2006), innovations in *VoV* (*ΔVoV*), and Chang, Christoffersen, and Jacobs (2013) innovations in market skewness factor (*ΔSKEW*) and market kurtosis factor (*ΔKURT)*. As expected, *MKT* is negatively correlated with both *ΔVIX* (-0.779) and *ΔVoV* (-0.044), supporting the leverage effect predicted by our model. Moreover, *VRP* is positively correlated with *ΔVoV* (0.145), consistent with our theory that the variance risk premium and the market volatility-of-volatility are both driven by the economic volatility-of-volatility. *ΔKURT* and *ΔSKEW* are highly correlated with a correlation value of -0.863, which is comparable to -0.83 reported by Chang, Christoffersen, and Jacobs (2013). *ΔVoV* shows little correlation with *ΔVIX* (0.049), *ΔSKEW* (-0.017), and *ΔKURT* (-0.010), which suggests that *ΔVoV* should be an independent state variable that cannot be explained by these market moments studied in the literature.

5. Pricing volatility-of-volatility risk in the cross-sectional stock returns

This section examines how market volatility-of-volatility risk affects cross-sectional average returns. Based on our market-based three-factor model with their empirical proxies, at the end of each month, we estimate the regression for each stock *i* using daily returns:

$$r_{i,t+1} - r_{f,t+1} = \alpha_i + \beta_{i,MKT} MKT_{t+1} + \beta_{i,VIX} \Delta VIX_{t+1} + \beta_{i,VoV} \Delta VoV_{t+1} + \varepsilon_{i,t+1}. \tag{33}$$

We construct a set of testing assets that are sufficiently disperse in exposure to aggregate volatility-of-volatility innovations by sorting firms on $\beta_{i,VoV}$ loadings over the past month using the regression (33) with daily data. Our empirical model is an extension of Ang, Hodrick, Xing, and Zhang (2006). Following their work, we run the regression for all common stocks on NYSE, AMEX, and NASDAQ with more than 17 daily observations. After the portfolio formation, we calculate the value-weighted daily and monthly stock returns for each portfolio. If market volatility-of-volatility is a priced risk factor, we should expect to see a monotonic decreasing pattern in the portfolio returns.

5.1. Portfolios sorted on market volatility-of-volatility risk

Table 2 provides the performance of portfolios sorted on $\beta_{i,VoV}$. Stocks are sorted into quintile portfolios based on $\beta_{i,VoV}$, from the lowest (quintile 1) to the highest (quintile 5). Consistent with the model, we find that stocks with positive return sensitivities to market volatility-of-volatility (quintile 5) have lower stock returns than stocks with negative return sensitivities (quintile 1) by 0.88 percent per month with t- statistic of -2.32. Controlling for Fama and French four factor model, the '5-1' long-short portfolio still gives a significant alpha of -0.96 percent per month with a *t*-statistic of -2.59.

To check whether our results are robust to firm characteristics, Table 3 shows performance of portfolios sorted on $\beta_{i,VoV}$, controlling for market capitalization (*Size*), book-to-market ratio (*B/M*), past 11-month returns (*RET_2_12*), and Amihud's illiquidity (*ILLIQ*), respectively. We first sort stocks into five quintiles based *Size*.

Then, within each quintile, we sort stocks based on their $\beta_{i,VoV}$ loadings into five portfolios. All portfolios are rebalanced monthly and are value weighted. The five portfolios sorted on $\beta_{i,VoV}$ are then averaged over each of the five *Size* sorted portfolios, resulting $\beta_{i,VoV}$ quintile portfolios controlling for *Size*. *B/M*, *RET_2_12*, and *ILLIQ* are analyzed with the same procedure as described above. The Fama and French four factor alpha of the '5-1' long–short portfolio remains significant controlling for these four variables, i.e. at -0.45 percent with a *t*-statistic of -2.14 controlling for *Size*, at -0.88 percent with a *t*-statistic of -3.15 controlling for *B/M*, at -0.52 percent with a *t*-statistic of -2.12 controlling for *RET_2_12*, and at -0.53 percent with a *t*-statistic of -2.30 controlling for *ILLIQ*. Hence, the low returns to high $\beta_{i,VoV}$ stocks are not completely driven by the existing well-known firm characteristics.

5.2. Portfolios sorted on market volatility risk

Table 4 provides the performance of portfolios sorted on $\beta_{i,VIX}$, using the same approach as on $\beta_{i,VoV}$. We find evidence consistent with Ang, Hodrick, Xing, and Zhang's (2006) findings that there is a significant difference of -0.87 percent per month with a *t*-statistic of -2.17 between the stock returns with high volatility risk and the stocks with low volatility risk. Controlling for Fama and French four factor model, the '5-1' long-short portfolio gives a significant alpha of -1.18 percent per month with a *t*-statistic of -3.24.

Table 5 considers two-way sorted portfolios on $\beta_{i,VIX}$ and $\beta_{i,VoV}$. We sort stocks into quintile portfolios based on $\beta_{i,VIX}$, from the lowest (quintile 1) to the highest (quintile 5), and independently sort stocks into quintile portfolios based on $\beta_{i,VoV}$. The five portfolios sorted on $\beta_{i,VIX}$ are then averaged over each of the five $\beta_{i,VoV}$ portfolios, resulting $\beta_{i,VIX}$ quintile portfolios controlling for $\beta_{i,VoV}$. Similar approach gives $\beta_{i,VoV}$ quintile portfolios controlling for $\beta_{i,VIX}$. Controlling for volatility risk loadings ($\beta_{i,VIX}$), we still find market volatility-of-volatility risk carries a statistically significant return differential of -0.97 percent per month with a *t*-statistic of -2.84. On the other hand, controlling for market volatility-of-volatility risk loadings ($\beta_{i,VoV}$), we find that the return difference between stocks with high volatility risk and stocks with low volatility risk is still large in magnitude, at -0.68 percent per month with *t*-value of -1.94. Thus, the valuation effect of market volatility-of-volatility risk is not affected after controlling for $\beta_{i,VIX}$, suggesting that the market volatility-of-volatility risk is a pricing factor independent with the aggregate volatility factor.

5.3. Portfolios sorted on market skewness risk

At the end of each month, we estimate the model of Chang, Christoffersen, and Jacobs (2013) with ex ante higher moments of market returns for each stock *i*:

$$\begin{aligned} r_{i,t+1} - r_{f,t+1} = \alpha_i + \beta_{i,MKT} MKT_{t+1} + \beta_{i,VIX} \Delta VIX_{t+1} \\ + \beta_{i,SKEW} \Delta SKEW_{t+1} + \beta_{i,KURT} \Delta KURT_{t+1} + \varepsilon_{i,t+1}. \end{aligned} \tag{34}$$

We sort stocks into quintile portfolios based on $\beta_{i,SKEW}$, from the lowest (quintile 1) to the highest (quintile 5), and we also independently sort stocks into quintile portfolios based on $\beta_{i,VoV}$.

Panel A of Table 6 provides the performance of portfolios sorted on $\beta_{i,SKEW}$. We find that there is a significant difference of -0.65 percent per month with a *t*-statistic of -1.99 between the stock returns with high skewness risk and the stocks with low skewness risk. Controlling for Fama and French four factor model, the '5-1' long-short portfolio gives a significant alpha of -0.75 percent per month with a *t*-statistic of -2.28.

Panel B shows the results for the $\beta_{i,SKEW}$ quintile portfolios controlling for $\beta_{i,VoV}$ quintile portfolios. Controlling for market volatility-of-volatility risk loadings ($\beta_{i,VoV}$), we find the market skewness risk premium is much weaker, carrying a statistically insignificant return differential of -0.33 percent per month with a t-statistic of -1.17. On the other hand, as reported in Panel C, controlling for market skewness risk loadings ($\beta_{i,VoV}$), we find that the '5-1' long-short portfolio still gives a significant return differential of -0.82 percent per month with a t-statistic of -2.38. In summary, the market skewness risk is less likely to explain the market volatility-of-volatility risk, whereas part of the skewness return differential can be explained by the market volatility-of-volatility risk.

5.4. Price of market volatility-of-volatility risk

We apply the two-pass regressions of Fama-MacBeth (1973) to estimate the price of market volatility-of-volatility risk. Our set of test assets are the 25 portfolios formed on intersection of $\beta_{i,VIX}$ quintile portfolios and $\beta_{i,VoV}$ quintile portfolios. For each portfolio, we estimate the time-series regression of equation (33) using the post-formation daily value-weighted portfolio returns to obtain the post-formation factor loadings. We then conduct the cross-sectional regression:

$$\mathbb{E}[r_p] - r_f = \lambda_{MKT}\beta_{p,MKT} + \lambda_{VIX}\beta_{p,VIX} + \lambda_{VoV}\beta_{p,VoV}. \tag{35}$$

The dependent variable is the monthly value-weighted portfolio return and the independent variables are the post-ranking return betas estimated from equation (33) using full-sample daily portfolio returns. Robust Newey and West (1987) t-statistics with six lags that account for autocorrelations are used. The cross-sectional regression gives the estimates of risk prices, i.e. λ_{MKT}, λ_{VIX}, and λ_{VoV}.

Panel A of Table 7 reports the estimate of risk prices from the 25 portfolios sorted on $\beta_{i,VIX}$ and $\beta_{i,VoV}$. In column [2], we find that λ_{VoV} is negative (-4.11) with a significant t-statistic of -3.27. Controlling for the market volatility risk, as reported in column [3], λ_{VoV} is still significantly negative (-3.66) with a t-statistic of -2.35, which accounts for -3.66×0.21= -0.77 percent per month of the '5-1' return in

Table 2. Controlling for all of the other factors, as shown in column [6], λ_{VoV} remains significantly negative (-3.62) with a t-statistic of -2.68, which accounts for -3.62×0.21= -0.76 percent. In contrast, λ_{VIX} is only significant in column [1], with t-value of -3.29. Thus, our empirical findings suggest that market volatility-of-volatility indeed is an independently priced risk factor relative to aggregate volatility factor.

In Panel B, we the estimate of risk prices from the 25 portfolios sorted on intersection of $\beta_{i,SKEW}$ quintile portfolios and $\beta_{i,VoV}$ quintile portfolios. Consequently, the testing assets are sufficiently disperse in the exposure to aggregate volatility-of-volatility as well as in the exposure to aggregate skewness. In column [2], we find that λ_{VoV} is negative (-2.07) with a significant t-statistic of -1.77. As shown in column [6], λ_{VoV} is significantly negative (-1.86) with a t-statistic of -1.75, whereas λ_{SKEW} is positive (2.76) with insignificant t-statistic of 1.36. Therefore, relative to the market skewness factor, the variance of market variance remains a priced risk factor.

5.5. Leverage effect, feedback effect and volatility-of-volatility risk premium

To further explore the mechanism that volatility-of-volatility risk affects asset prices, we investigate whether the volatility-of-volatility risk contributes to the feedback effect. To identify the timely volatility-of-volatility shocks, at the end of each day, we estimate the regression of equation (33) using daily stock returns over the past 22 days. We then sort stocks into quintile portfolios on the estimated $\beta_{i,VoV}$ for each day and calculate the event-time daily value-weighted portfolio returns ranging from -11 to 11 in days.

If market volatility-of-volatility is priced, an anticipated increase in market volatility-of-volatility raises the required rate of return, implying an immediate stock price decline and higher future returns. As shown in Figure 2, consistent with the channel of feedback effect, stocks with negative return sensitivities to market volatility-of-volatility have lower returns before the portfolio formation and earn higher post-formation returns than stocks with positive return sensitivities.

We construct a portfolio that is long the lowest quintile and short the highest quintile and we denote the portfolio as low-minus-high. The low-minus-high portfolio has, by construction, large negative exposure to innovations in market volatility-of-volatility. The theory of the leverage effect and the feedback effect predict an asymmetric cross-correlation between the aggregate volatility and the pre-formation and the post-formation low-minus-high returns.

As can be seen in the top panel of Figure 3, the pre-formation low-minus-high returns are negatively correlated with *VIX* measured at the portfolio formation date while the correlations between *VIX* and the post-formation low-minus-high returns are positive, supporting the leverage effect and the feedback effect associated with the aggregate volatility.

Moreover, our theory for the leverage effect and the feedback effect similarly predicts an asymmetric cross-correlation between market volatility-of-volatility and the pre-formation and the post-formation low-minus-high returns. As can be seen in the bottom panel of Figure 3, the low-minus-high return is negatively correlated with *VoV* at the portfolio formation date while the correlation between *VoV* and one-day post-formation low-minus-high return is positive. The market volatility-of-volatility carries a negative contemporaneous correlation of -0.232, which is much larger in magnitude than -0.057 for the contemporaneous leverage effect associated with market volatility. The correlation between one-day post-formation low-minus-high return and market volatility-of-volatility is 0.100, which is larger than 0.060 for the correlation between the return and the market volatility. The stronger asymmetric cross-correlation, despite less persistent, supports the leverage effect and the feedback effect associated with market volatility-of-volatility. Therefore, market volatility-of-volatility seems to be the state variable that determines the time-varying risk premium.

5.6. Robustness to volatility spreads

In this section, we check whether our results are robust to existing well-known volatility spreads that affect cross-sectional stock returns. We construct the implied-realized volatility spread (*IVOL-TVOL*), which is, as described in Bali and Hovakimian (2009), defined as the average of implied volatilities by at-the-money call and put minus the total volatility calculated using daily returns in the previous month; the call-put implied volatility spread (*CIVOL-PIVOL*), which is, as described in Bali and Hovakimian (2009) and Yan (2011), defined as the at-the-money call implied volatility minus the at-the-money put implied volatility; the expected individual variance risk premium (*EVRP*), which is, as described in the data section and in Han and Zhou (2012), defined as the difference between the model-free implied variance and the five-minute realized variance. Since we extract the volatility data from OptionMetrics Volatility Surface file as Yan (2011) do, we choose the 30-day maturity put and call options with deltas equal to -0.5 and 0.5, respectively.

Panel A of Table 8 shows the performance of portfolios sorted on each of the volatility spreads as well as on $\beta_{i,VoV}$ using stocks with available equity options. The Fama and French four factor alpha of the '5-1' long–short portfolio is 0.63 percent with a *t*-statistic of 1.76 for *IVOL-TVOL* quintile portfolios, 1.66 percent with a *t*-statistic of 6.70 for *CIVOL-PIVOL* quintile portfolios, 0.96 percent with a *t*-statistic of -2.21 for *EVRP* quintile portfolios, and -0.85 percent with a *t*-statistic of -2.38 for $\beta_{i,VoV}$ quintile portfolios. Hence, our results for market volatility-of-volatility risk remain significant in the options market and consistent with the literature, all of the three volatility variables carry significant premium in the cross-section.

We construct two-way sorted portfolios formed on intersection of each of the volatility spread quintile portfolios and $\beta_{i,VoV}$ quintile portfolios. Panel B shows the results for the $\beta_{i,VoV}$ quintile portfolios controlling for volatility spread quintile portfolios. The Fama and French four factor alpha of the '5-1' long–short portfolio remains significant controlling for these three variables, i.e. at -0.76 percent with a *t*-statistic of -2.38 controlling for *IVOL-TVOL*, at -0.74 percent with a *t*-statistic of -2.29 controlling for *CIVOL-PIVOL*, and at -0.66 percent with a *t*-statistic of -2.20 controlling for *EVRP*. Hence, the low returns to high $\beta_{i,VoV}$ stocks are not driven by the existing well-known volatility spreads.

As shown by Yan (2011), *CIVOL-PIVOL* is proxy for a disaster type jump risk that affects the cross-sectional stock returns. Our empirical finding that the pricing of $\beta_{i,VoV}$ is robust to *CIVOL-PIVOL* provides indirect evidence that the market volatility-of-volatility risk cannot be completely explained by a peso-problem like jump risk.

5.7. Robustness to firm-level Fama-MacBeth regressions

In this section, we examine whether the pricing of market volatility-of-volatility risk is robust to the firm-level analysis. We employ individual stocks as the set of test assets to avoid potentially spurious results that could arise when the test portfolios are constructed toward a specific model (Lewellen, Nagel, and Shanken, 2010). Furthermore, a stock-level analysis could increase the power of the test by controlling for several individual characteristics at the same time.

We test our market-based three factor model at firm-level with the following cross-sectional regression:

$$\begin{aligned} r_{i,t+1} - r_{f,t+1} = c_0 + \lambda_{MKT}\beta_{i,MKT,t} + \lambda_{VIX}\beta_{i,VIX,t} + \lambda_{VoV}\beta_{i,VoV,t} \\ + c_{FIRM}\, FirmCharac_{i,t} + c_{VOL}\, VolatilityCharac_{i,t} + \varepsilon_{i,t+1}, \end{aligned} \tag{36}$$

where the dependent variable is the monthly individual stock returns; $\beta_{i,MKT,t}$, $\beta_{i,VIX,t}$, and $\beta_{i,VoV,t}$ are post-ranking betas estimated from the same 25 portfolios in section 5.4 formed on intersection of $\beta_{i,VIX}$ quintile portfolios and $\beta_{i,VoV}$ quintile portfolios; $FirmCharac_{i,t}$ consists of *Size*, *B/M*, *RET_2_12*, and *ILLIQ*; and $VolatilityCharac_{i,t}$ includes *IVOL-TVOL*, *CIVOL-PIVOL*, and *EVRP*. Robust Newey and West (1987) *t*-statistics with six lags that account for autocorrelations are used. Following the methodology of Fama and French (1992), we assign each of the 25 portfolio-level post-ranking beta estimates to the individual stocks within the portfolio at that time. Thus, individual stock betas vary over time because the portfolio compositions change each month.

Table 9 reports the results from the firm-level Fama-MacBeth regressions. In column [2], we find that λ_{VoV} is negative (-3.07) with a significant *t*-statistic of -4.07. Controlling for the market volatility risk, as reported in column [3], λ_{VoV} is still significantly negative (-3.07) with a *t*-statistic of -4.17, which accounts for -3.07×0.21= -0.65 percent per month of the '5-1' return in Table 2. Controlling for all of the other variables, as shown in column [6], λ_{VoV} remains significantly negative (-3.12) with a *t*-statistic of -2.57, which accounts for -3.62×0.21= -0.66 percent. Thus, the firm-level evidence confirms our results that the market volatility-of-volatility is a priced risk factor in the cross-sectional stock returns.

6. Pricing market volatility-of-volatility in the cross-sectional variance risk premium

The second test in this paper is to examine whether market volatility-of-volatility is priced in the cross-sectional variance risk premium. For each stock with available equity options in each day, we calculate the 30-day model-free implied variance ($IV_{i,t+1}$). Then, at the end of each month, we estimate the variance beta with respect to market volatility-of-volatility ($\beta_{i,VoV}^V$) for each stock by regressing the stock's $IV_{i,t+1}$ on VoV_{t+1} over the past month. We use $\beta_{i,VoV}^V$ to construct a set of test portfolios. Our theory suggests that the cross-sectional expected variance risk premium is determined by:

$$EVRP_p \equiv IV_p - ERV_p = -\lambda_{VIX}^V \beta_{p,VIX}^V - \lambda_{VoV}^V \beta_{p,VoV}^V. \tag{37}$$

We estimate $\beta_{p,VIX}^V$ and $\beta_{p,VoV}^V$ by the following time-series regression:

$$IV_{p,t+1} = \alpha_p^V + \beta_{p,VIX}^V \Delta\widetilde{VIX}_{t+1}^2 + \beta_{p,VoV}^V \Delta VoV_{t+1} + \varepsilon_{p,t+1}^V, \tag{38}$$

where $IV_{p,t+1}$ is the post-formation portfolio implied variance; $\Delta\widetilde{VIX}_{t+1}^2$ as the innovations in market variance, which is measured as the ARMA(1,1) model residuals of squared VIX divided by 12, orthogonalized by ΔVoV_{t+1}. While we define $\Delta VIX_{t+1} = VIX_{t+1} - VIX_t$ for the stock return beta as in Ang, Hodrick, Xing, and Zhang (2006) for the compatibility, our variance beta is estimated by $\Delta\widetilde{VIX}_{t+1}^2$ for the model consistency.

Table 10 provides the performance of portfolios sorted on $\beta_{i,VoV}^V$. Stocks are sorted into quintile portfolios based on $\beta_{i,VoV}^V$, from the lowest (quintile 1) to the highest (quintile 5). After the portfolio formation, we calculate monthly value-weighted expected variance risk premium and daily value-weighted model-free implied variance for each portfolio. Consistent with the model, we find that stocks with high variance sensitivities to market volatility-of-volatility (quintile 5) have higher expected variance risk premium than stocks with low variance sensitivities (quintile 1) by 67.7 (in percentages squared) per month with t- statistic of -5.15. The magnitude of the cross-sectional difference in variance risk premium is large compared to the market variance risk premium, which is 17.3 (in percentages squared) per month during our sample period. Panel B reports the performance of portfolios sorted on $\beta_{i,VIX}^V$. The results are very similar to the portfolios sorted on $\beta_{i,VoV}^V$. In fact, we find that the cross-sectional Spearman correlation between $\beta_{i,VoV}^{V}$ and $\beta_{i,VIX}^{V}$ is 0.99, which is also part of the reason why we use the orthogonalized innovations in market variance as risk factor for individual variance.

We estimate the price of risks in cross-sectional *EVRP* using the 25 portfolios sorted on $\beta_{i,VoV}^{V}$. We apply the two-pass regressions of Fama-MacBeth (1973) to estimate the price of market volatility-of-volatility risk in *EVRP*. After the portfolio formation, we calculate monthly value-weighted expected variance risk premium, daily value-weighted model-free implied variance, and daily value-weighted stock returns for each portfolio. In the first stage, for each portfolio, we estimate the post-ranking variance betas by equation (38) using daily portfolio implied variance. For the second stage, we regress the cross-sectional monthly portfolio *EVRP* on variance betas obtained from the first stage, using Fama–MacBeth (1973) cross-sectional regression by equation (37).

Table 11 reports the estimate of risk prices in *EVRP* from the 25 portfolios sorted on $\beta_{i,VoV}^{V}$. In column [2], we find that $-\lambda_{VoV}^{V}$ is positive (5.99) with a significant *t*-statistic of 5.32. Controlling for the market volatility risk ($\beta_{p,VIX}^{V}$), as reported in column [3], $-\lambda_{VoV}^{V}$ is still significantly positive (5.20) with a *t*-statistic of 4.04, which accounts for 5.20×8.32= 43.3 (in percentages squared) per month of the '5-1' *EVRP* in Table 10. Controlling for all of the other factors, as shown in column [6], $-\lambda_{VoV}^{V}$ remains significantly positive (1.53) with a *t*-statistic of 2.27, which accounts for 1.53×8.32= 12.7 (in percentages squared). Thus, our empirical findings suggest that market volatility-of-volatility is priced risk factor in the cross-sectional variance risk premium.

7. Return predictability

In this section, we check the return predictability afforded by market volatility-of-volatility. The theoretical model suggests that market volatility-of-volatility is positively related to economic volatility-of-volatility. Hence, we should expect that our *VoV* measure can predict future stock returns as market variance risk premium does.

Panel A of Table 12 reports the estimates of the one-period return predictability regression using daily S&P 500 logarithmic returns multiplied by 22 on the lagged variance risk premium (*VRP*), market volatility-of-volatility (*VoV*), and innovations in market skewness (*ΔSKEW*). Robust Newey and West (1987) *t*-statistics with sixteen lags that account for autocorrelations are used. Consistent with the theory, we find that *VoV* positively predicts one-period ahead market return in all of the specifications. In panel B, we use the monthly S&P 500 logarithmic returns as the dependent variable, and the independent variables are sampled at the end of the previous month. Robust Newey and West (1987) *t*-statistics with six lags are used. The predictability afforded by *VoV* remains significant.

Overall, the return predictability supports the volatility-of-volatility feedback effect implied by our model. The evidence for the predictability afforded by the market volatility-of-volatility suggests that economic volatility-of-volatility is an important state variable that affects the aggregate asset prices.

8. Conclusions

Market volatility-of-volatility appears to be a state variable that is important for asset pricing. We develop a market-based three-factor model, in which market risk, market volatility risk, and market volatility-of-volatility risk determine the cross-sectional asset prices. We find that market volatility-of-volatility risk is priced in the cross-sectional stock returns. Stocks with negative larger return exposure to market volatility-of-volatility have substantially higher future stock returns, even after we account for exposures to the Fama and French four factors, market skewness factor, firm characteristics and volatility characteristics. We also find that market volatility-of-volatility risk is priced in the cross-sectional variance risk premium.

Our measure of market volatility-of-volatility generates leverage effect and feedback effect. Stocks with negative larger return exposure to market volatility-of-volatility have substantially lower contemporaneous stock returns, which suggests that market volatility-of-volatility is priced such that an anticipated increase in market volatility-of-volatility risk raises the required rate of return, leading to an immediate stock price decline and higher future returns. Our evidence on return predictability for the aggregate market portfolio supports feedback effect implied by our model. The predictability evidence afforded by the market volatility-of-volatility also suggests that economic volatility-of-volatility is an important state variable.

Our study shows that market volatility-of-volatility risk affects the cross-sectional expected variance risk premium. One direction for future research is to explore whether market volatility-of-volatility risk plays a role in tradable volatility-related assets such as equity option returns or index option returns. Future research could also investigate whether our measure of market volatility-of-volatility affects the VIX option returns.

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Table 1 Properties of the daily factors.

We report summary statistics and Spearman correlations for the daily factors, including Fama and French (1993) four factors (*MKT*, *SMB*, *HML*, and *UMD*), the market variance risk premium (*VRP*), the *VIX* index, our measure of variance of market variance (*VoV*), and Chang, Christoffersen, and Jacobs (2013) market skewness factor (*SKEW*) and market kurtosis factor (*KURT*). *ΔVIX* is the first difference of *VIX*. *ΔVoV*, *ΔSKEW*, and *ΔKURT* are the residuals from fitting an ARMA(1,1) regression using *VoV*, *SKEW*, and *KURT*, respectively. The sample period is from January 1996 to December 2010.

	Panel A: Summary statistics								
	MKT(%)	*SMB(%)*	*HML(%)*	*UMD(%)*	*VRP(%)*	*VIX(%)*	*VoV(%)*	*SKEW*	*KURT*
Mean	0.023	0.010	0.016	0.024	17.260	23.098	0.054	-1.663	9.313
Median	0.070	0.030	0.020	0.070	15.082	22.150	0.003	-1.637	8.672
Std.Dev.	1.300	0.629	0.682	1.035	21.182	9.509	0.563	0.485	3.466

	Panel B: Spearman correlation								
	MKT	*SMB*	*HML*	*UMD*	*VRP*	*ΔVIX*	*ΔVoV*	*ΔSKEW*	*ΔKURT*
MKT	1.000								
SMB	0.038	1.000							
HML	-0.279	-0.082	1.000						
UMD	-0.047	0.053	-0.078	1.000					
VRP	-0.222	-0.050	0.006	0.062	1.000				
ΔVIX	-0.779	0.031	0.210	0.020	0.180	1.000			
ΔVoV	-0.044	-0.003	-0.038	0.036	0.145	0.049	1.000		
ΔSKEW	-0.237	-0.022	0.026	0.034	0.076	0.248	-0.017	1.000	
ΔKURT	0.311	0.014	-0.057	-0.017	-0.106	-0.307	-0.010	-0.863	1.000

Table 2 Portfolios sorted on $\boldsymbol{\beta_{i,VoV}}$.

At the end of each month, we run the following regression for each stock using daily returns:

$$r_{i,t+1} - r_{f,t+1} = \alpha_i + \beta_{i,MKT} MKT_{t+1} + \beta_{i,VIX} \Delta VIX_{t+1} + \beta_{i,VoV} \Delta VoV_{t+1} + \varepsilon_{i,t+1}.$$

We sort stocks into quintile portfolios based on $\beta_{i,VoV}$, from the lowest (quintile 1) to the highest (quintile 5). After the portfolio formation, we calculate the value-weighted daily and monthly stock returns for each portfolio. The column "5-1" refers to the hedge portfolio that longs portfolio 5 and shorts portfolio 1. For each portfolio, we estimate the same time-series regression as above using the post-formation daily portfolio returns to obtain the post-formation factor loadings. We compute the risk-adjusted return of each portfolio with respect to Fama-French four factors (*MKT*, *SMB*, *HML*, and *UMD*) from the intercept estimate of a time-series regression of the monthly portfolio returns on the four factors. Numbers in parentheses are *t*-statistics. *Size* reports the average market capitalization (in billion) for firms within the portfolio; *B/M* reports the average book-to-market ratios; *RET_2_12* reports the average of past 11-month returns prior to last month; *ILLIQ* reports the average of Amihud's (2002) illiquidity measure. The sample period is from January 1996 to December 2010.

	Portfolios ranking					
	1	*2*	*3*	*4*	*5*	*5-1*
Risk-adjusted performance of $\beta_{i,VoV}$ *sorted portfolios (monthly return)*						
Excess return	0.90	0.64	0.40	0.34	0.02	-0.88
	(1.61)	(1.70)	(1.20)	(0.92)	(0.04)	(-2.32)
α-CAPM	0.34	0.23	0.03	-0.07	-0.54	-0.89
	(1.21)	(2.01)	(0.33)	(-0.57)	(-2.54)	(-2.32)
α-FF3	0.28	0.21	0.02	-0.07	-0.54	-0.82
	(1.13)	(1.89)	(0.25)	(-0.67)	(-2.59)	(-2.18)
α-FF4	0.44	0.22	0.01	-0.08	-0.53	-0.96
	(1.88)	(1.99)	(0.05)	(-0.74)	(-2.50)	(-2.59)
Post-formation factor loadings (daily return)						
$\beta_{p,MKT}$	1.35	1.00	0.90	0.96	1.32	-0.02
	(78.37)	(125.34)	(145.90)	(127.57)	(79.40)	(-0.91)
$\beta_{p,VIX}$	0.06	-0.01	-0.02	-0.01	0.06	-0.01
	(5.34)	(-1.42)	(-4.70)	(-2.48)	(4.86)	(-0.41)
$\beta_{p,VoV}$	-0.04	-0.03	0.01	0.02	0.16	0.21
	(-1.93)	(-3.00)	(1.47)	(2.27)	(7.35)	(5.95)
Pre-formation characteristics						
Size($b)	1.03	2.87	3.49	3.24	1.33	0.30
B/M	1.19	0.88	0.84	0.83	1.10	-0.09
RET_2_12	12.31	14.96	15.11	14.37	11.76	-0.56
ILLIQ(10^6)	9.04	3.06	2.47	3.05	8.87	-0.17
Pre-formation factor loadings						
$\beta_{i.MKT}$	1.23	0.97	0.91	0.99	1.28	0.05
$\beta_{i.VIX}$	0.06	0.00	-0.01	-0.01	0.01	-0.05
$\beta_{i.VoV}$(10^2)	-6.23	-2.18	-0.15	1.86	5.69	11.92

Table 3 Portfolios sorted on $\boldsymbol{\beta_{i,VoV}}$, controlling for Size, B/M, momentum, and illiquidity.

This table shows performance of portfolios sorted on $\beta_{i,VoV}$, controlling for market capitalization (*Size*), book-to-market ratio (*B/M*), past 11-month returns (*RET_2_12*), and Amihud's illiquidity (*ILLIQ*), respectively. We first sort stocks into five quintiles based on their market capitalization (*Size*). Then, within each quintile, we sort stocks based on their $\beta_{i,VoV}$ loadings into five portfolios. All portfolios are rebalanced monthly and are value weighted. The five portfolios sorted on $\beta_{i,VoV}$ are then averaged over each of the five *Size* portfolios, resulting $\beta_{i,VoV}$ quintile portfolios controlling for *Size*. We compute the risk-adjusted return of each portfolio with respect to Fama-French four factors (*MKT*, *SMB*, *HML*, and *UMD*). *B/M*, *RET_2_12*, and *ILLIQ* are analyzed with the same procedure as described above. Numbers in parentheses are *t*-statistics.

	Portfolios ranking on $\beta_{i,VoV}$						*α-FF4*
	1	*2*	*3*	*4*	*5*	*5-1*	*5-1*
Controlling for Size	0.81	0.94	0.76	0.75	0.39	-0.42	-0.45
	(1.30)	(2.12)	(2.00)	(1.80)	(0.70)	(-1.93)	(-2.14)
Controlling for B/M	1.10	0.71	0.49	0.47	0.29	-0.81	-0.88
	(2.19)	(1.95)	(1.48)	(1.31)	(0.59)	(-2.81)	(-3.15)
Controlling for RET_2_12	0.53	0.52	0.41	0.29	0.04	-0.49	-0.52
	(1.05)	(1.21)	(0.99)	(0.69)	(0.07)	(-2.01)	(-2.12)
Controlling for ILLIQ	0.79	0.81	0.75	0.67	0.28	-0.51	-0.53
	(1.38)	(2.09)	(2.24)	(1.84)	(0.56)	(-2.18)	(-2.30)

Table 4 Portfolios sorted on $\boldsymbol{\beta_{i,VIX}}$.

At the end of each month, we run the following regression for each stock using daily returns:

$$r_{i,t+1} - r_{f,t+1} = \alpha_i + \beta_{i,MKT} MKT_{t+1} + \beta_{i,VIX} \Delta VIX_{t+1} + \beta_{i,VoV} \Delta VoV_{t+1} + \varepsilon_{i,t+1}.$$

We sort stocks into quintile portfolios based on $\beta_{i,VIX}$, from the lowest (quintile 1) to the highest (quintile 5). After the portfolio formation, we calculate the value-weighted daily and monthly stock returns for each portfolio. The column "5-1" refers to the hedge portfolio that longs portfolio 5 and shorts portfolio 1. For each portfolio, we estimate the same time-series regression as above using the post-formation daily portfolio returns to obtain the post-formation factor loadings. We compute the risk-adjusted return of each portfolio with respect to Fama-French four factors (*MKT*, *SMB*, *HML*, and *UMD*) from the intercept estimate of a time-series regression of the monthly portfolio returns on the four factors. Numbers in parentheses are *t*-statistics. *Size* reports the average market capitalization (in billion) for firms within the portfolio; B/M reports the average book-to-market ratios; *RET_2_12* reports the average of past 11 month returns prior to last month; *ILLIQ* reports the average of Amihud's (2002) illiquidity measure. The sample period is from January 1996 to December 2010.

	Portfolios ranking					
	1	*2*	*3*	*4*	*5*	*5-1*
Risk-adjusted performance of $\beta_{i,VIX}$ sorted portfolios (monthly return)						
Excess return	0.76	0.58	0.37	0.38	-0.11	-0.87
	(1.55)	(1.69)	(1.12)	(0.95)	(-0.18)	(-2.17)
α-CAPM	0.25	0.20	0.00	-0.07	-0.72	-0.96
	(1.17)	(1.99)	(0.01)	(-0.70)	(-2.49)	(-2.46)
α-FF3	0.30	0.23	-0.03	-0.11	-0.79	-1.09
	(1.41)	(2.52)	(-0.40)	(-1.20)	(-3.41)	(-2.99)
α-FF4	0.41	0.25	-0.05	-0.14	-0.77	-1.18
	(2.06)	(2.77)	(-0.60)	(-1.52)	(-3.27)	(-3.24)
Post-formation factor loadings (daily return)						
$\beta_{p,MKT}$	1.20	0.91	0.89	1.05	1.47	0.27
	(82.06)	(132.82)	(145.66)	(141.97)	(79.37)	(11.04)
$\beta_{p,VIX}$	0.02	-0.03	-0.03	0.01	0.12	0.10
	(1.62)	(-7.08)	(-7.37)	(1.44)	(9.25)	(6.00)
$\beta_{p,VoV}$	0.08	0.00	-0.03	-0.02	0.07	-0.02
	(4.28)	(0.29)	(-4.19)	(-2.35)	(2.73)	(-0.48)
Pre-formation characteristics						
Size($b)	1.28	3.39	3.63	2.67	0.99	-0.29
B/M	1.13	0.88	0.82	0.83	1.19	0.07
RET_2_12	10.88	14.82	15.39	15.02	12.33	1.46
ILLIQ(10^6)	9.42	2.76	2.28	2.72	9.31	-0.12
Pre-formation factor loadings						
$\beta_{i.MKT}$	-0.08	0.53	0.95	1.52	2.75	2.83
$\beta_{i.VIX}$	-1.23	-0.40	0.03	0.48	1.40	2.63
$\beta_{i,VoV}$(10^2)	-0.32	-0.17	0.00	0.18	-0.10	0.22

Table 5 Two-way sorted portfolios on $\boldsymbol{\beta_{i,VIX}}$ and $\boldsymbol{\beta_{i,VoV}}$.

At the end of each month, we run the following regression for each stock using daily returns:

$$r_{i,t+1} - r_{f,t+1} = \alpha_i + \beta_{i,MKT} MKT_{t+1} + \beta_{i,VIX} \Delta VIX_{t+1} + \beta_{i,VoV} \Delta VoV_{t+1} + \varepsilon_{i,t+1}.$$

We sort stocks into quintile portfolios based on $\beta_{i,VIX}$, from the lowest (quintile 1) to the highest (quintile 5), and independently sort stocks into quintile portfolios based on $\beta_{i,VoV}$. All portfolios are rebalanced monthly and are value weighted. The five portfolios sorted on $\beta_{i,VIX}$ are then averaged over each of the five $\beta_{i,VoV}$ portfolios, resulting $\beta_{i,VIX}$ quintile portfolios controlling for $\beta_{i,VoV}$. Similar approach gives $\beta_{i,VoV}$ quintile portfolios controlling for $\beta_{i,VIX}$. The column "5-1" refers to the hedge portfolio that longs portfolio 5 and shorts portfolio 1. For each portfolio, we estimate the same time-series regression as above using the post-formation daily portfolio returns to obtain the post-formation factor loadings. We compute the risk-adjusted return of each portfolio with respect to Fama-French four factors (*MKT*, *SMB*, *HML*, and *UMD*). Numbers in parentheses are *t*-statistics. Panel A and Panel B present the results for $\beta_{i,VIX}$ quintile portfolios and $\beta_{i,VoV}$ quintile portfolios, respectively. In Panel C, we report the monthly value-weighted portfolio excess return for each of the 25 portfolios. The sample period is from January 1996 to December 2010.

	Portfolios ranking					
	1	*2*	*3*	*4*	*5*	*5-1*
	Panel A: Ranking on $\beta_{i,VIX}$, controlling for $\beta_{i,VoV}$					
Excess return	0.66	0.58	0.48	0.40	-0.02	-0.68
	(1.34)	(1.50)	(1.30)	(0.90)	(-0.03)	(-1.94)
α-FF4	0.25	0.20	0.03	-0.17	-0.70	-0.95
	(1.38)	(2.14)	(0.38)	(-1.49)	(-3.37)	(-2.97)
	Panel B: *Ranking on $\beta_{i,VoV}$, controlling for $\beta_{i,VIX}$*					
Excess return	0.90	0.65	0.35	0.28	-0.08	-0.97
	(1.57)	(1.56)	(0.94)	(0.70)	(-0.14)	(-2.84)
α-FF4	0.39	0.18	-0.09	-0.20	-0.66	-1.05
	(1.82)	(1.50)	(-0.91)	(-1.83)	(-3.43)	(-3.16)
	Panel C: $\beta_{i,VIX} \times \beta_{i,VoV}$ two-way sorted portfolios					
	Ranking on $\beta_{i,VoV}$					
Ranking on $\beta_{i,VIX}$	*1*	*2*	*3*	*4*	*5*	*5-1*
1	0.71	1.02	0.79	0.62	0.16	-0.56
	(1.12)	(2.01)	(1.62)	(1.23)	(0.23)	(-1.09)
2	1.06	0.46	0.61	0.46	0.31	-0.75
	(1.86)	(1.28)	(1.75)	(1.21)	(0.55)	(-1.51)
3	1.32	0.68	0.30	0.30	-0.18	-1.50
	(2.35)	(1.86)	(0.90)	(0.83)	(-0.37)	(-3.16)
4	1.10	0.64	0.32	0.29	-0.35	-1.45
	(1.70)	(1.53)	(0.81)	(0.67)	(-0.61)	(-3.01)
5	0.30	0.45	-0.29	-0.25	-0.31	-0.61
	(0.42)	(0.65)	(-0.52)	(-0.42)	(-0.43)	(-1.50)
5-1	-0.41	-0.57	-1.08	-0.87	-0.47	-0.06
	(-0.83)	(-1.11)	(-2.18)	(-1.77)	(-0.95)	(-0.10)

Table 6 Two-way sorted portfolios on $\boldsymbol{\beta_{i,SKEW}}$ and $\boldsymbol{\beta_{i,VoV}}$

At the end of each month, we separately run the following regressions for each stock using daily returns:

$$r_{i,t+1} - r_{f,t+1} = \alpha_i + \beta_{i,MKT} MKT_{t+1} + \beta_{i,VIX} \Delta VIX_{t+1} + \beta_{i,SKEW} \Delta SKEW_{t+1} + \beta_{i,KURT} \Delta KURT_{t+1} + \varepsilon_{i,t+1}$$

$$r_{i,t+1} - r_{f,t+1} = \alpha_i + \beta_{i,MKT} MKT_{t+1} + \beta_{i,VIX} \Delta VIX_{t+1} + \beta_{i,VoV} \Delta VoV_{t+1} + \varepsilon_{i,t+1}.$$

We sort stocks into quintile portfolios based on $\beta_{i,SKEW}$, from the lowest (quintile 1) to the highest (quintile 5), and independently sort stocks into quintile portfolios based on $\beta_{i,VoV}$. All portfolios are rebalanced monthly and are value weighted. The five portfolios sorted on $\beta_{i,SKEW}$ are then averaged over each of the five $\beta_{i,VoV}$ portfolios, resulting $\beta_{i,SKEW}$ quintile portfolios controlling for $\beta_{i,VoV}$. Similar approach gives $\beta_{i,VoV}$ quintile portfolios controlling for $\beta_{i,SKEW}$. The column "5-1" refers to the hedge portfolio that longs portfolio 5 and shorts portfolio 1. For each portfolio, we estimate the same time-series regression as above using the post-formation daily portfolio returns to obtain the post-formation factor loadings. We compute the risk-adjusted return of each portfolio with respect to Fama-French four factors (*MKT*, *SMB*, *HML*, and *UMD*). Numbers in parentheses are *t*-statistics. Panel A presents the results for the $\beta_{i,SKEW}$ quintile portfolios. Panel B shows the results for the $\beta_{i,SKEW}$ quintile portfolios controlling for $\beta_{i,VoV}$ quintile portfolios while Panel C shows the results for the $\beta_{i,VoV}$ quintile portfolios controlling for $\beta_{i,SKEW}$ quintile portfolios. In Panel D, we report the monthly value-weighted portfolio excess return for each of the 25 portfolios. The sample period is from January 1996 to December 2010.

	Portfolios ranking					
	1	*2*	*3*	*4*	*5*	*5-1*
			Panel A: Ranking on $\beta_{i,SKE\square}$			
Excess return	0.88	0.43	0.40	0.37	0.23	-0.65
	(1.67)	(1.14)	(1.21)	(1.00)	(0.43)	(-1.99)
α-FF4	0.41	0.04	0.02	-0.08	-0.34	-0.75
	(1.98)	(0.35)	(0.23)	(-0.77)	(-1.70)	(-2.28)
			Panel B: Ranking on $\beta_{i,SKEW}$, controlling for $\beta_{i,VoV}$			
Excess return	0.66	0.52	0.35	0.55	0.33	-0.33
	(1.23)	(1.22)	(0.94)	(1.32)	(0.62)	(-1.17)
α-FF4	0.18	0.05	-0.11	0.08	-0.27	-0.45
	(0.93)	(0.50)	(-1.30)	(0.74)	(-1.51)	(-1.59)
			Panel C: Ranking on $\beta_{i,VoV}$, controlling for $\beta_{i,SKEW}$			
Excess return	0.88	0.67	0.38	0.43	0.05	-0.82
	(1.51)	(1.63)	(1.03)	(1.08)	(0.09)	(-2.38)
α-FF4	0.31	0.23	-0.06	-0.05	-0.51	-0.82
	(1.46)	(2.12)	(-0.58)	(-0.47)	(-2.48)	(-2.44)

Table 6 (continued.)

	Panel D: $\beta_{i,SKEW} \times \beta_{i,VoV}$ two-way sorted portfolios					
	Ranking on $\beta_{i,VoV}$					
Ranking on $\beta_{i,SKEW}$	*1*	*2*	*3*	*4*	*5*	*5-1*
1	0.96	0.80	0.38	1.00	0.19	-0.76
	(1.33)	(1.38)	(0.72)	(1.81)	(0.30)	(-1.48)
2	1.33	0.73	0.29	0.06	0.18	-1.15
	(2.12)	(1.80)	(0.76)	(0.14)	(0.33)	(-2.17)
3	0.36	0.66	0.35	0.43	-0.03	-0.39
	(0.64)	(1.79)	(1.07)	(1.13)	(-0.05)	(-0.82)
4	1.15	0.48	0.66	0.48	-0.03	-1.18
	(1.91)	(1.24)	(1.78)	(1.20)	(-0.06)	(-2.70)
5	0.58	0.67	0.25	0.21	-0.06	-0.64
	(0.85)	(1.28)	(0.48)	(0.39)	(-0.08)	(-1.27)
5-1	-0.37	-0.12	-0.13	-0.79	-0.25	0.12
	(-0.75)	(-0.33)	(-0.28)	(-1.78)	(-0.53)	(0.18)

Table 7 The price of volatility-of-volatility risk

Panel A reports the Fama–MacBeth (1973) factor premiums on 25 portfolios sorted on intersection of $\beta_{i,VIX}$ quintile portfolios and $\beta_{i,VoV}$ quintile portfolios, using our market-based three factors (*MKT*, *ΔVIX*, and *ΔVoV*), Chang, Christoffersen, and Jacobs (2013) market skewness factor (*ΔSKEW*), and Fama-French four factors (*MKT*, *SMB*, *HML*, and *UMD*). We estimate the first stage return betas using the daily full-sample post-formation value-weighted returns. Then, we regress the cross-sectional monthly portfolio returns on daily return betas from the first stage, using Fama–MacBeth (1973) cross-sectional regression. Panel B reports the Fama–MacBeth (1973) factor premiums on 25 portfolios sorted on intersection of $\beta_{i,SKEW}$ quintile portfolios and $\beta_{i,VoV}$ quintile portfolios. Robust Newey and West (1987) *t*-statistics with six lags that account for autocorrelations are reported in parentheses. The sample period is from January 1996 to December 2010.

	Fama-MacBeth cross-sectional regressions					
	[1]	[2]	[3]	[4]	[5]	[6]
	Panel A: 25 portfolios sorted on $\beta_{i,VIX} \times \beta_{i,VoV}$ (5×5)					
MKT	0.54	0.53	0.57	0.55	0.55	0.55
	(1.34)	(1.28)	(1.41)	(1.37)	(1.36)	(1.36)
ΔVIX	-5.31		-3.14	-3.87	-3.97	-5.00
	(-3.29)		(-1.61)	(-0.94)	(-0.95)	(-1.14)
ΔVoV		-4.11	-3.66	-3.84	-3.88	-3.62
		(-3.25)	(-2.35)	(-2.68)	(-2.66)	(-2.68)
SMB				-0.93	-0.94	-0.94
				(-1.25)	(-1.29)	(-1.28)
HML				-0.27	-0.17	-0.45
				(-0.40)	(-0.27)	(-0.66)
UMD				-1.73	-1.65	-1.02
				(-1.58)	(-1.47)	(-0.96)
ΔSKEW					0.52	0.49
					(0.50)	(0.48)
ΔKURT						-15.54
						(-1.13)
Adj. R^2	0.13	0.10	0.18	0.24	0.24	0.25

Table 7 (continued.)

	Fama-MacBeth cross-sectional regressions					
	[1]	[2]	[3]	[4]	[5]	[6]
Panel B: 25 portfolios sorted on $\beta_{i,SKEW} \times \beta_{i,VoV}$ (5×5)						
MKT	0.48	0.45	0.61	0.43	0.64	0.64
	(1.16)	(1.12)	(1.57)	(1.06)	(1.64)	(1.63)
ΔVIX		-0.31	4.51	-0.79	8.47	8.46
		(-0.17)	(1.27)	(-0.45)	(1.73)	(1.62)
ΔVoV	-2.00	-2.07	-1.76		-1.87	-1.86
	(-1.84)	(-1.77)	(-1.77)		(-1.86)	(-1.75)
SMB			-0.31		-0.51	-0.50
			(-0.37)		(-0.60)	(-0.58)
HML			-1.40		-0.87	-0.87
			(-2.30)		(-1.32)	(-1.24)
UMD			1.70		2.33	2.33
			(1.09)		(1.44)	(1.43)
ΔSKEW				2.73	2.77	2.76
				(2.53)	(1.45)	(1.36)
ΔKURT				5.69		0.58
				(0.59)		(0.05)
Adj. R^2	0.10	0.15	0.26	0.13	0.26	0.26

Table 8 Two-way portfolios sorted on volatility spreads and $\beta_{i,VoV}$

This table shows performance of portfolios sorted on implied-realized volatility spread (*IVOL-TVOL*), the call-put implied volatility spread (*CIVOL-PIVOL*), the expected individual variance risk premium (*EVRP*), and $\beta_{i,VoV}$ using stocks with available equity options. We independently sort stocks into quintile portfolios based on each of the four variables, from the lowest (quintile 1) to the highest (quintile 5). All portfolios are rebalanced monthly and are value weighted. We compute the risk-adjusted return of each portfolio with respect to Fama-French four factors (*MKT*, *SMB*, *HML*, and *UMD*). Panel A reports the results for the one-way sorted portfolios. We construct two-way sorted portfolios formed on intersection of each of the volatility spread quintile portfolios and $\beta_{i,VoV}$ quintile portfolios. Panel B shows the results for the $\beta_{i,VoV}$ quintile portfolios controlling for volatility spread quintile portfolios. Numbers in parentheses are *t*-statistics. The sample period is from January 1996 to December 2010.

	Portfolios ranking						*α-FF4*
	1	*2*	*3*	*4*	*5*	*5-1*	*5-1*
Panel A: One-way sorted portfolios							
Ranking on $\beta_{i,VoV}$	0.88	0.64	0.45	0.24	0.15	-0.73	-0.85
	(1.68)	(1.80)	(1.31)	(0.63)	(0.28)	(-2.02)	(-2.38)
Ranking on IVOL-TVOL	-0.04	0.45	0.51	0.89	0.73	0.77	0.63
	(-0.08)	(1.17)	(1.49)	(2.38)	(1.47)	(2.10)	(1.76)
Ranking on CIVOL-PIVOL	-0.24	0.17	0.44	0.69	1.15	1.39	1.66
	(-0.51)	(0.47)	(1.20)	(1.83)	(2.33)	(5.21)	(6.70)
Ranking on EVRP	-0.24	0.29	0.55	0.73	0.93	1.16	0.96
	(-0.39)	(0.76)	(1.50)	(1.65)	(1.41)	(2.50)	(2.21)
Panel B: Two-way sorted portfolios, ranking on $\beta_{i,VoV}$							
Controlling for IVOL-TVOL	0.97	0.77	0.45	0.32	0.28	-0.69	-0.76
	(1.74)	(2.00)	(1.21)	(0.82)	(0.53)	(-2.11)	(-2.38)
Controlling for CIVOL-PIVOL	0.82	0.60	0.50	0.31	0.16	-0.66	-0.74
	(1.54)	(1.54)	(1.40)	(0.78)	(0.29)	(-2.03)	(-2.29)
Controlling for EVRP	0.72	0.66	0.55	0.38	0.13	-0.59	-0.66
	(1.33)	(1.52)	(1.33)	(0.87)	(0.25)	(-1.96)	(-2.20)

Table 9 Firm-level Fama-MacBeth regressions

This table reports the results for the firm-level Fama-MacBeth regressions. We run the following cross-sectional regression:

$$r_{i,t+1} - r_{f,t+1} = c_0 + \lambda_{MKT}\beta_{i,MKT,t} + \lambda_{VIX}\beta_{i,VIX,t} + \lambda_{VoV}\beta_{i,VoV,t}$$
$$+ c_{FIRM}\ FirmCharac_{i,t} + c_{VOL}\ VolatilityCharac_{i,t} + \varepsilon_{i,t+1},$$

where the dependent variable is the monthly individual stock returns; $\beta_{i,MKT,t}$, $\beta_{i,VIX,t}$, and $\beta_{i,VoV,t}$ are post-ranking betas estimated from the 25 portfolios formed on intersection of $\beta_{i,VIX}$ quintile portfolios and $\beta_{i,VoV}$ quintile portfolios; $FirmCharac_{i,t}$ consists of *Size*, *B/M*, *RET_2_12*, and *ILLIQ*; $VolatilityCharac_{i,t}$ includes *IVOL-TVOL*, *CIVOL-PIVOL*, and *EVRP*. Following the methodology of Fama and French (1992), we assign each of the 25 portfolio-level post-ranking beta estimates to the individual stocks within the portfolio at that time. Robust Newey and West (1987) *t*-statistics with six lags that account for autocorrelations are reported in parentheses. The sample period is from January 1996 to December 2010.

	Fama-MacBeth regressions: individual stocks					
	[1]	[2]	[3]	[4]	[5]	[6]
Intercept	-2.14	-2.95	-2.58	1.33	1.39	1.39
	(-1.56)	(-3.86)	(-1.95)	(1.03)	(1.06)	(0.99)
logSize	-0.05	-0.05	-0.05	-0.04	-0.04	-0.10
	(-0.78)	(-0.75)	(-0.82)	(-0.53)	(-0.54)	(-1.06)
logBM	0.26	0.27	0.26	0.15	0.14	0.06
	(1.75)	(1.76)	(1.74)	(0.99)	(0.95)	(0.36)
RET_2_12	0.21	0.22	0.22	0.32	0.32	0.29
	(0.57)	(0.61)	(0.60)	(0.75)	(0.74)	(0.63)
ILLIQ	0.02	0.02	0.02	-0.04	-0.04	-2.76
	(3.51)	(3.54)	(3.53)	(-0.29)	(-0.26)	(-0.29)
$\beta_{i,MKT}$	2.47	3.30	2.93	-0.69	-0.72	-0.78
	(1.74)	(4.15)	(2.10)	(-0.50)	(-0.52)	(-0.53)
$\beta_{i,VIX}$	1.50		1.54	8.32	8.43	6.30
	(0.36)		(0.39)	(1.90)	(1.90)	(1.33)
$\beta_{i,VoV}$		-3.07	-3.07	-3.04	-2.96	-3.12
		(-4.07)	(-4.17)	(-2.74)	(-2.67)	(-2.57)
IVOL-TVOL				0.69	0.70	0.90
				(2.16)	(2.21)	(2.75)
CIVOL-PIVOL					5.31	6.43
					(8.09)	(6.70)
EVRP						0.09
						(2.22)
Adj. R^2	0.04	0.04	0.05	0.07	0.08	0.09
No. obs.	824,428	824,428	824,428	310,221	310,221	241,096

Table 10 Portfolios sorted on $\boldsymbol{\beta_{i,VoV}^{V}}$

We sort stocks into quintile portfolios based on $\beta_{i,VoV}^{V}$, from the lowest (quintile 1) to the highest (quintile 5). After the portfolio formation, we calculate the value-weighted daily 30-day model-free implied variance and monthly 30-day variance risk premium for each portfolio. The column "5-1" refers to the hedge portfolio that longs portfolio 5 and shorts portfolio 1. For each portfolio, we estimate the post-ranking variance betas by running the following regression using daily portfolio implied variance:

$$\Delta IV_{p,t+1} = \alpha_p^V + \beta_{p,VIX}^V \Delta\widetilde{VIX}_{t+1}^2 + \beta_{p,VIX}^V \Delta VoV_{t+1} + \varepsilon_{p,t+1}^V.$$

Numbers in parentheses are *t*-statistics. The sample period is from January 1996 to December 2010.

	Portfolios ranking					
	1	*2*	*3*	*4*	*5*	*5-1*
			Panel A: Ranking on $\beta_{i,VoV}^{V}$			
EVRP(%^{2})	6.6	23.6	36.5	46.3	74.2	67.7
	(4.24)	(9.62)	(9.83)	(7.29)	(5.35)	(5.15)
Post-formation daily variance beta						
$\beta_{p,VIX}^{V}$	0.48	0.75	1.12	1.47	2.45	1.97
	(94.24)	(73.65)	(62.04)	(39.01)	(43.77)	(35.80)
$\beta_{p,VoV}^{V}$	2.73	4.12	3.35	5.71	11.06	8.32
	(21.96)	(16.52)	(7.62)	(6.18)	(8.09)	(6.19)
Volatility characteristics						
IV	65.0	115.6	180.5	284.0	534.4	469.4
ERV	58.4	92.0	143.9	237.7	460.1	401.7
			Panel B: Ranking on $\beta_{i,VIX}^{V}$			
EVRP(%^{2})	6.6	23.5	36.9	47.4	75.5	68.9
	(4.23)	(9.66)	(9.76)	(7.42)	(5.37)	(5.19)
Post-formation daily variance beta						
$\beta_{p,VIX}^{V}$	0.49	0.78	1.09	1.56	2.39	1.90
	(97.63)	(82.77)	(61.10)	(39.04)	(45.45)	(36.95)
$\beta_{p,VoV}^{V}$	2.86	3.79	4.17	5.00	10.44	7.58
	(23.53)	(16.46)	(9.60)	(5.13)	(8.13)	(6.02)
Volatility characteristics						
IV	64.9	115.8	180.9	284.8	542.6	477.6
ERV	58.3	92.2	144.0	237.4	467.0	408.7

Table 11 The price of volatility-of-volatility risk in cross-sectional *EVRP*

This table reports the Fama–MacBeth (1973) factor premiums on 25 portfolios sorted on $\beta^V_{i,VoV}$, using our market-based three factors (*MKT*, *ΔVIX*, and *ΔVoV*), Chang, Christoffersen, and Jacobs (2013) market skewness factor (*ΔSKEW*), and Fama-French four factors (*MKT*, *SMB*, *HML*, and *UMD*). For each portfolio, we estimate the post-ranking variance betas by running the following regression using daily portfolio implied variance:

$$\Delta IV_{p,t+1} = \alpha^V_p + \beta^V_{p,VIX}\Delta\widetilde{VIX}^2_{t+1} + \beta^V_{p,VIX}\Delta VoV_{t+1} + \varepsilon^V_{p,t+1}.$$

Then, we regress the cross-sectional monthly portfolio expected variance risk premium on the post-ranking variance betas using Fama–MacBeth (1973) cross-sectional regression:

$$EVRP_p = -\lambda^V_{VIX}\beta^V_{p,VIX} - \lambda^V_{VoV}\beta^V_{p,VoV}.$$

In column from 4 to 6, we include the post-ranking return betas obtained from running regression using daily portfolio returns on the risk factors:

$$EVRP_p = -\lambda^V_{VIX}\beta^V_{p,VIX} - \lambda^V_{VoV}\beta^V_{p,VoV} + \lambda_{MKT}\beta_{p,MKT} + \lambda_{SMB}\beta_{p,SMB} + \lambda_{HML}\beta_{p,HML} + \lambda_{UMD}\beta_{p,UMD} + \lambda_{SKEW}\beta_{p,SKEW} + \lambda_{KURT}\beta_{p,KURT}.$$

Robust Newey and West (1987) *t*-statistics with six lags that account for autocorrelations are reported in parentheses. The sample period is from January 1996 to December 2010.

	Fama-MacBeth cross-sectional regressions					
	[1]	[2]	[3]	[4]	[5]	[6]
	25 portfolios sorted on $\beta^V_{i,VoV}$					
$\beta^V_{p,VIX}$	34.28		5.29	-34.11	-21.22	-20.66
	(5.55)		(1.27)	(-5.43)	(-3.77)	(-3.67)
$\beta^V_{p,VoV}$		5.99	5.20	2.74	1.62	1.53
		(5.32)	(4.04)	(4.54)	(2.36)	(2.27)
$\beta_{p,MKT}$						31.20
						(4.58)
$\beta_{p,SMB}$				9.51	80.73	71.21
				(0.72)	(4.53)	(4.29)
$\beta_{p,HML}$				45.15	82.22	63.51
				(2.03)	(3.46)	(3.16)
$\beta_{p,UMD}$				-140.27	-47.56	-64.82
				(-6.34)	(-1.86)	(-2.27)
$\beta_{p,SKEW}$					-100.91	-151.05
					(-6.76)	(-7.23)
$\beta_{p,KURT}$					-528.89	-211.78
					(-3.19)	(-1.06)
Adj. R^2	0.34	0.39	0.55	0.73	0.77	0.77

Table 12 Return predictability regressions

Panel A reports the estimates of the one-period return predictability regression using daily market return on the lagged variance risk premium (*VRP*), variance of market variance (*VoV*), market skewness (*SKEW*), and market kurtosis (*KURT*). In panel B, we use the monthly market return as the dependent variable, and the independent variables are sampled at the end of the previous month. We multiply Daily market return in Panel A is multiplied by 22. Robust Newey and West (1987) *t*-statistics with sixteen lags in Panel A and with six lags in Panel B that account for autocorrelations are reported in parentheses. The sample period is from January 1996 to December 2010.

	Dependent variable= MKT (t)					
	[1]	[2]	[3]	[4]	[5]	[6]
			Panel A: Daily return regressions			
Intercept	-2.214	-0.970	0.123	-2.302	0.577	0.674
	(-2.82)	(-1.50)	(0.26)	(-2.36)	(0.30)	(0.35)
VRP (t-1)	0.153			0.140	0.142	0.128
	(3.81)			(3.86)	(3.89)	(3.45)
VIX (t-1)		0.027		0.000	-0.001	-0.003
		(1.95)		(0.00)	(-0.06)	(-0.15)
VoV (t-1)			5.406	4.939	4.980	5.054
			(2.47)	(2.12)	(2.16)	(2.19)
SKEW (t-1)					2.378	2.304
					(1.50)	(1.46)
KURT(t-1)					0.121	0.136
					(0.53)	(0.60)
MKT (t-1)						-0.041
						(-2.23)
Adj. R^2	0.012	0.002	0.011	0.021	0.021	0.023
			Panel B: Monthly return regressions			
Intercept	-0.369	0.604	0.280	-0.183	0.796	0.630
	(-1.11)	(1.13)	(0.68)	(-0.41)	(0.59)	(0.49)
VRP (t-1)	0.045			0.041	0.041	0.039
	(5.47)			(4.68)	(4.24)	(3.82)
VIX (t-1)		-0.004		-0.004	-0.004	0.001
		(-0.34)		(-0.59)	(-0.62)	(0.18)
VoV (t-1)			1.682	1.246	1.352	1.460
			(3.06)	(2.44)	(2.61)	(2.53)
SKEW (t-1)					0.008	0.004
					(0.52)	(0.28)
KURT(t-1)					0.000	0.000
					(0.21)	(-0.23)
MKT (t-1)						0.124
						(1.52)
Adj. R^2	0.047	-0.004	0.009	0.044	0.036	0.041

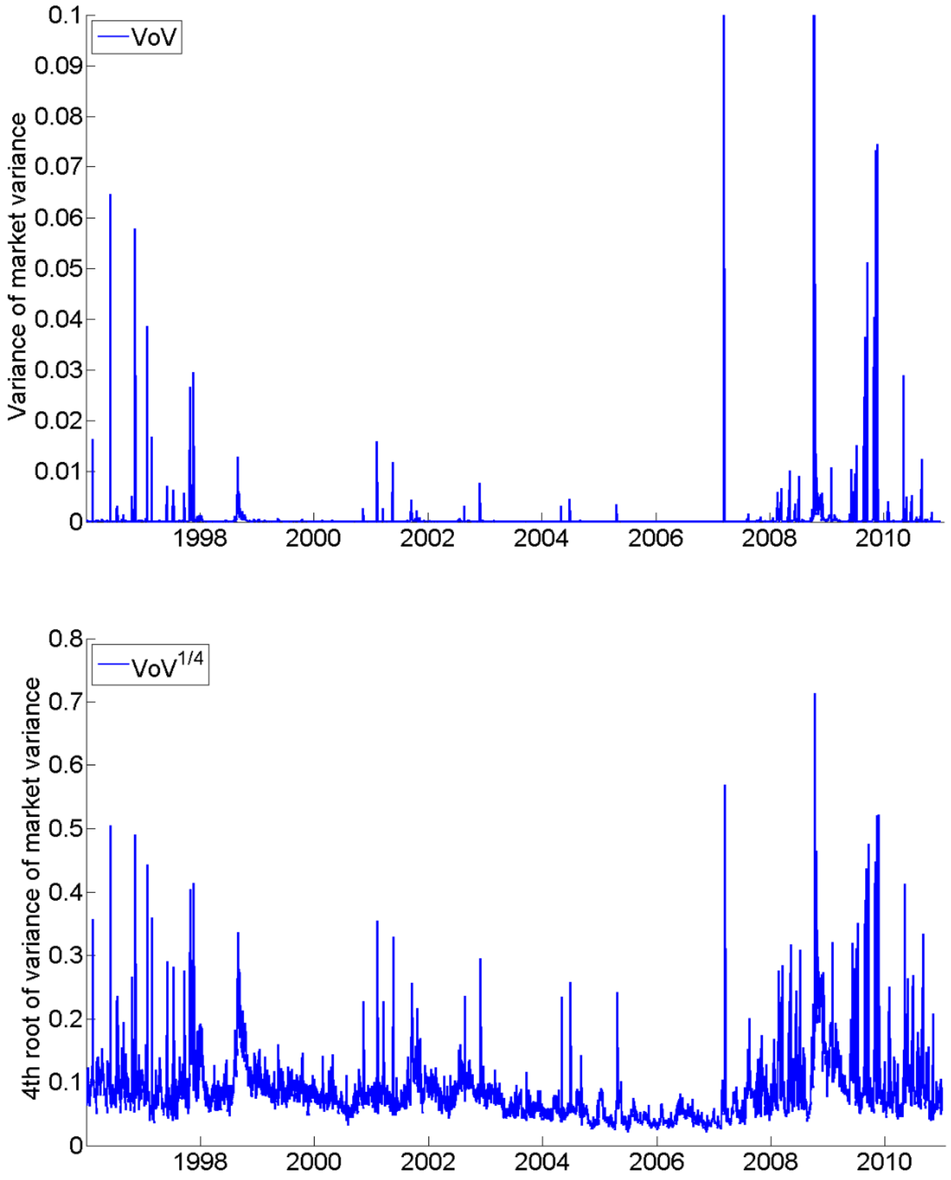


Figure 1. Daily market volatility-of-volatility (*VoV*). We plot daily market volatility-of-volatility over the time period January 1996 through December 2010. We partition the tick-by-tick S&P500 index options data into five-minute intervals, and then we estimate the model-free implied variance for each interval. For each day, we use the bipower variation formula on the five-minute based annualized 30-day model-free implied variance estimates within the day, resulting in our daily measure of market volatility-of-volatility (*VoV*).

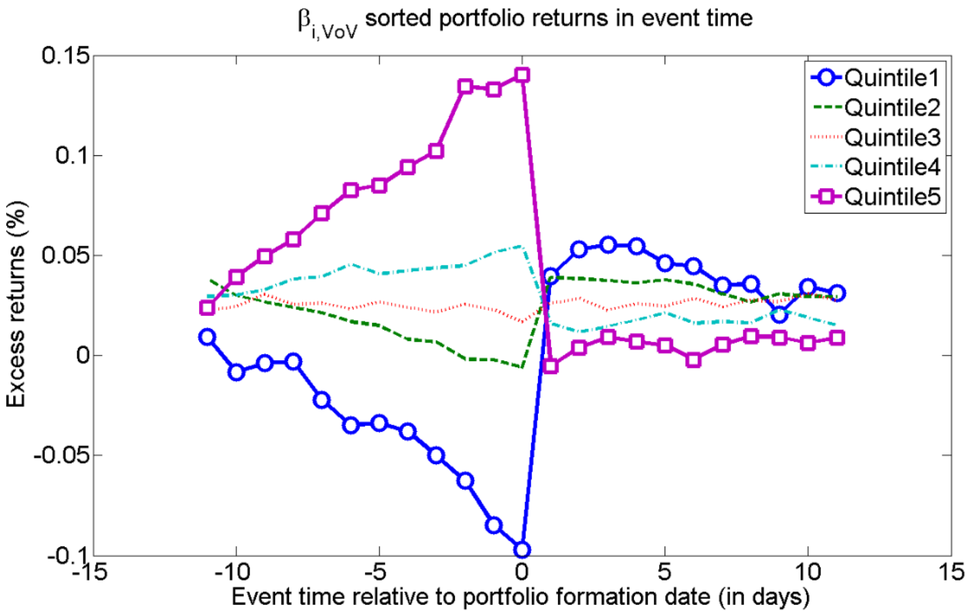
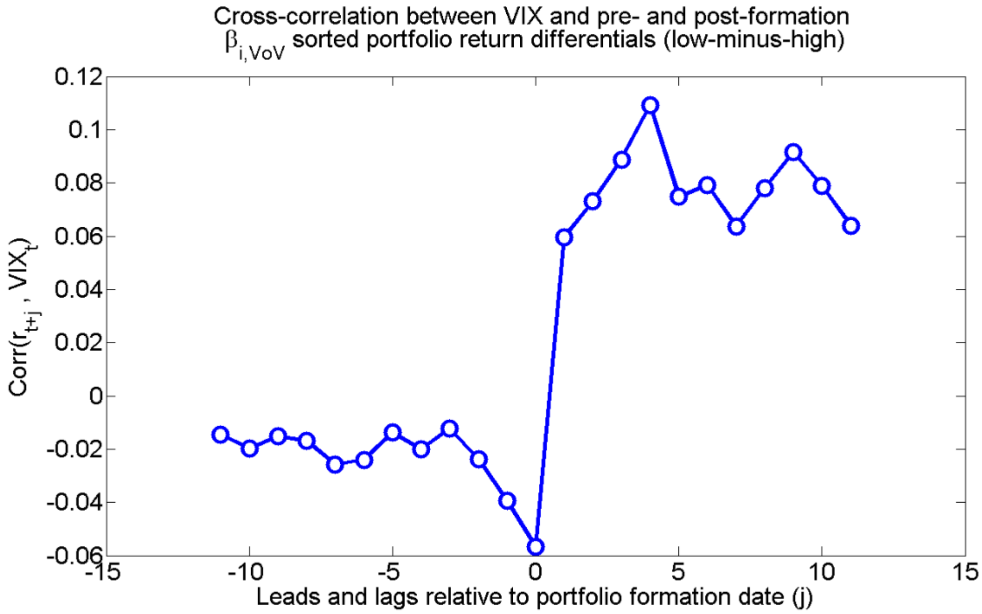


Figure 2. Performance of portfolios sorted on $\boldsymbol{\beta_{i,VoV}}$ in event time. At the end of each day, we estimate the regression of equation (33) using daily stock returns over the past 22 days. We then sort stocks into quintile portfolios on the estimated $\beta_{i,VoV}$ for each day and calculate the event-time daily value-weighted portfolio returns ranging from -11 to 11 in days.



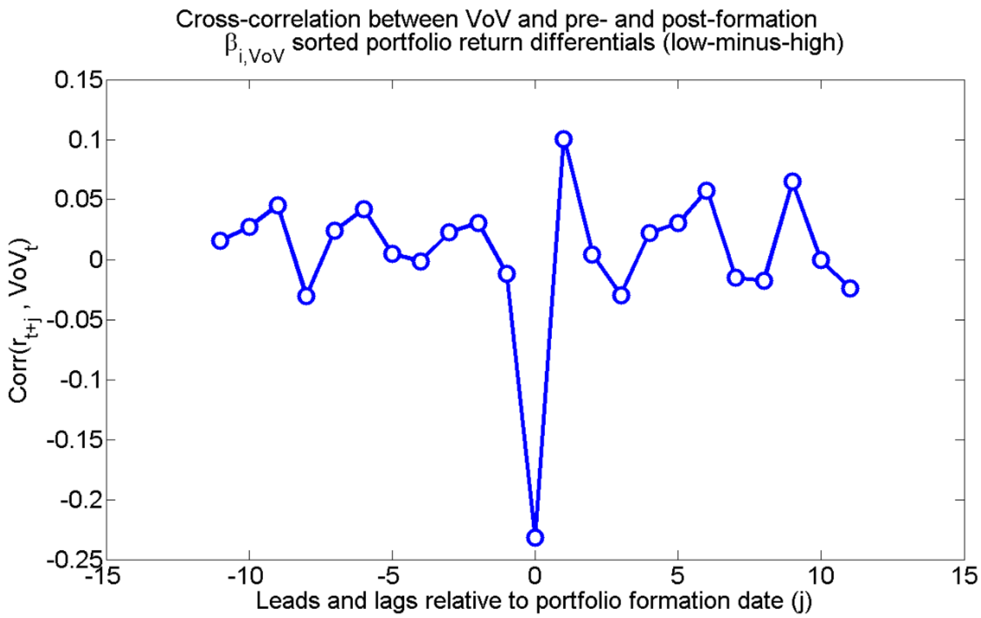


Figure 3. Cross-correlations. The plots are based on the pre-formation and post-formation of quintile portfolio return differentials (low-minus-high; long the lowest quintile and short the highest quintile) formed on $\beta_{i,VoV}$. The top panel shows the sample cross-correlation between the *VIX* and portfolio formation time leads and lags of the low-minus-high ranging from -11 to 11 days. The bottom panel shows the sample cross-autocorrelations between the market volatility-of-volatility (*VoV*) and the returns.